# CP Algebra 2

## Unit 2-1: Factoring and Solving Quadratics

### NOTE PACKET

<table>
<thead>
<tr>
<th>Name: ____________________________</th>
<th>Period ______</th>
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</table>

### Learning Targets:

**Factoring Quadratic Expressions**

0. I can add, subtract and multiply polynomial expressions

1. I can factor using GCF.

2. I can factor by grouping.

3. I can factor when \( a \) is one.

4. I can factor when \( a \) is not equal to one.

5. I can factor perfect square trinomials.

6. I can factor using difference of squares.

**Solving Quadratic Equations**

7. I can solve by factoring.

8. I can solve by taking the square root.

9. I can perform operations with imaginary numbers.

10. I can solve by completing the square.

11. I can solve equations using the quadratic formula (with rationalized denominators).

12. I can use the discriminant to determine the number and type of solutions.

13. I can write quadratic equations given the real solutions.
Operations on Polynomials

Date: ____________

After this lesson and practice, I will be able to ...

☐ Add, subtract, and multiply polynomials (LT 0)

To put an expression in standard form, simply put in __________ order of powers.

The degree is equal to the ________________.

Ex. \(2x^5 - 6x^4 - 9x^3 + x^2 - 4\) is in standard form. The degree is _____.

<table>
<thead>
<tr>
<th>Put in standard form</th>
<th>Degree</th>
</tr>
</thead>
<tbody>
<tr>
<td>(descending order)</td>
<td>(highest exp.)</td>
</tr>
</tbody>
</table>

1) \(-2x + 4x^2 + 1 + 8x^3\)

2) \(x + 3x^5 - 2\)

To add & subtract polynomials, simply combine like terms (CLT).

Ex. \(7x^3 + 2x^3 + 4x = \) ________________ \(\text{ Notice, we do NOT add the exponents.}\)

4) \((5x^2 - 4x - 8) + (2x^2 + 3x + 6) =\)

5) \((50x^2 - 12x - 3) - (8x^3 - 9x^2 - 2x + 4) =\)

To multiply polynomials, use the F.O.I.L. method or BOX method.

6) \((x - 4)(x + 6)\)

7) \((5x + 3)(8x - 4)\)

8) \((3x - 5)^2\)

9) \(8x(2x^2 - 3x + 4)\)

10) \((2x - 7)(3x^2 + x - 3)\)

11) \((3x - 1)^3\)
Factoring Basics

After this lesson and practice, I will be able to ...

☐ factor using GCF. (LT 1) factor by grouping. (LT 2)
☐ factor when a is one. (LT 3) factor when a is not one. (LT 4)

When we’ve modeled real-life situations with functions before, it’s been with LINEAR functions (example: at-bats vs. runs in baseball, GB vs. cost of a data plan, etc.).

Suppose we wanted to determine how high you could throw a tennis ball? What would the path of the ball look like?

As you can see, in order to model situations such as these, we require a function family that is not LINEAR. For situations such as these, we can use the QUADRATIC family of functions.

A Quadratic Function has a degree of 2.

(Throughout the year, we will be studying several different families of functions. We will learn how to graph and solve each type of function. We’re going to start Quadratics with solving!)

**Quadratic Function:**
A function that can be written in the form __________________________ where ____________.

**Standard Form of a Quadratic EQUATION:**

You might recognize these from previous math classes! Try to solve a quadratic equation using the usual method we used to solve linear equations …

\[ x^2 - 8x + 7 = -5 \]

It turns out we’ll need some new (and some not so new) skills in order to solve quadratic equations. One of those essential skills is **factoring**.

To **factor completely** is to write a polynomial as a product of __________________________ polynomials with integer coefficients. Make sure your final answers contain only unfactorable pieces (PRIME).

*We will factor polynomials of all degrees not just quadratic of degree 2.*

**Factored Form of a Quadratic EQUATION:**
**Factor Using Greatest Common Factor (GCF) (LT 1)**

The 1st step in factoring any expression is to always look for the greatest common factor that will go into each term evenly. This will be the GREATEST number that will divide into ALL coefficients & the VARIABLES that are common to ALL terms. If the variables have exponents, you factor out the HIGHEST exponent that they ALL have in common which is the lowest exponent.

**Example 1:** Factor completely, If not factorable, write PRIME.

A) \(14x^2 - 21x\)  
B) \(80n^7 + 8n^4 + 16n^3\)  
C) \(6x^5 y - 2y\)

D) \(5(x+1) - 3y(x+1)\) \hspace{1cm} This is called a common BINOMIAL factor.  
E) \(3a(b - 2) - 4a^5(b - 2)\)

While the following technique does not necessarily apply exclusively to QUADRATIC expressions, it will provide us the skills necessary to factor quadratics.

**Factor By Grouping (LT 2)**

*Factor by grouping* - pairing together binomials that have common monomial factors:

\[
ra + rb + sa + sb = r(a + b) + s(a + b) = (r + s)(a + b)
\]

When a polynomials contains 4 terms, it can often be factored by grouping (FBG).

**Ex)** Factor: \(5x + 5y + ax + ay\)

a) Group by pairs and pull out GCF  \hspace{1cm} \rightarrow \hspace{1cm} 5( ) + a( )

b) Pull out common binomial  \hspace{1cm} \rightarrow \hspace{1cm} ( ) \hspace{1cm} ( )

then multiply times the sum or diff of 2 gcfs \hspace{1cm} common binomial \hspace{1cm} gcf + gcf

**Example 2:** Factor completely, If not factorable, write PRIME.

A) \(9x + 3y - 3bx - by\)  
B) \(5x^3 - 35x^2 + 2x - 14\)
Example 3: Factor completely, If not factorable, write PRIME.

A) \(3n^3 - 18n^2 - 2n + 12\)  
B) \(x^3 - 6x^2 + 7x - 42\)

B) \(9r^3 + 15r^2 - 3r - 5\)  
C) \(u^4 + u^2v^2 - 7u^2 - 7v^2\)

Factoring when a is one (LT 3)

You probably remember doing this kind of factoring in Algebra 1 and/or Geometry.

Recall:
If the constant is a negative, then we need \(________________________\) (________________________).  
If the constant is a positive, then we need either \(_______________\) or \(_______________\).

The \(_______________\) term which tell us which one! \(\text{Same sign sum... Different signs difference}\)

Write in \(Ax^2 + Bx + C\) form - Look for the factors that multiple to \(AC\) and add/subtract \(B\).
Split the middle term and FBG.

Example 4: Factor completely, If not factorable, write PRIME. (Remember GCF first!)

A) \(x^2 - 11x + 10\)  
B) \(x^2 - 4x - 21\)  
C) \(x^3 + 2x^2 - 15x\)

Example 5: Factor completely, If not factorable, write PRIME.

A) \(x^2 - 17x + 72\)  
B) \(x^2 - 8x + 12\)
Example 5: Factor completely, If not factorable, write PRIME. (continued)
C) \(4x^2 + 20x - 24\)  
D) \(6x^5 + 30x^4 + 36x^3\)

Factoring when \(a\) is not one (LT 4)
Example 6: Factor completely, If not factorable, write PRIME.
A) \(7x^2 - 43x + 6\)  
B) \(12x^2 + 37x + 11\)
C) \(3x^2 + 11x + 8\)  
D) \(7x^2 + 37x - 30\)

Example 7: Factor completely, If not factorable, write PRIME. If expressions are factored more than once, write ALL the factors as your final product.
A) \(18x^2 - 90x + 100\)  
B) \(6x^2 - 21x - 27\)
C) \(15y^2 - 8y + 1\)  
D) \(5y^2 - 25y - 30\)
More Factoring

After this lesson and practice, I will be able to ...

☐ factor perfect square trinomials. (LT 5)
☐ factor using difference of squares. (LT 6)

When factoring, there are types of polynomials that are so common that it is helpful to know the patterns so you can identify the factors quickly.

Factoring Perfect Square Trinomials (LT 5)

Let's start by factoring \(x^2 + 14x + 49\) using our familiar methods:

**Observe:**
- The last term is a _____________ ____________. The first term is a ____________ ____________.
- The middle term is __________ the product of the quantities being squared in the ____ and _____ terms.
- ____________ is positive

\[
x^2 + 2xy + y^2 = ( x + y )^2
\]
\[
x^2 - 2xy + y^2 = ( x - y )^2
\]

If the A and C product is large, check for Perfect Square Trinomial

**Example 1:** Determine if the quadratic is a perfect square trinomial.
A) \(x^2 + 50x + 100\) B) \(x^2 - 6x + 9\) C) \(16x^2 + 24x + 9\)

**Example 2:** Factor completely the perfect square trinomials from above.
A) [Blank] B) [Blank] C) [Blank]

**Example 3:** Factor completely using the perfect square trinomial pattern.
A) \(9x^2 - 24x + 16\) C) \(25r^2 + 30r + 9\)
Factoring Using Difference of Two Squares (LT 6)

Let’s start by factoring \( x^2 - 25 \) using our familiar methods: What are A, B, and C in this problem?

Observe:
- The last term is a ______________ ______________. The first term is a ______________ ______________.

- The two factors are identical except for the ______________.
- These 2 factors are called CONJUGATES

\[
x^2 - y^2 =
\]

but \( x^2 + y^2 \) is PRIME under real numbers

Example 4: Factor completely. If not factorable, write prime
A) \( x^2 - 121 \)  
B) \( 4x^2 - 49 \)  
C) \( x^2 + 4 \)

Example 5: Factor completely, If not factorable, write PRIME.
A) \( 16x^2 - 9 \)  
B) \( 27m^2 - 75 \)  
C) \( 16x^4 - 81 \)
D) \( (a+5)^2 - m^2 \)  
E) \( y^2 - (9 + w)^2 \)  
F) \( 2b^3 - 200b \)

G) \( 25 - x^2 \)  
H) \( x^2 - 7 \)  
I) \( x^2 + 100 \)
Solving Quadratic Equations

After this lesson and practice, I will be able to ...

☐ solve by factoring. (LT 7)
☐ solve by taking the square root. (LT 8)

It’s finally time to start solving! The first way of solving combines factoring and the zero product property. Here’s what that property states:

If \( ab = 0 \), then ________________________________.

You’ll first want to get your standard form equation set equal to _______. Then get it in factored form. To get an equation in factored form, just write it in standard form, then factor! 😊

Once the quadratic equation is in factored form, use the zero product property. Essentially to do this, you just try to find the value for \( x \) that makes the equation ________________.

Example 1: Solve by factoring.
A) \((x + 4)(3x - 2) = 0\)
B) \(2x - 6x^2 = 0\)

B) \(x^2 - x - 12 = 0\)
C) \(2(x - 5)(3x + 7) = 0\)

D) \(x(8x + 1)(2x - 9) = 0\)
E) \(2n^2 - 13n + 23 = 3\)

*F) \(15x^6 + 12x^5 - 105x^4 - 84x^3 = 0\)
Sometimes you won’t be able to factor as nicely as you did above. What’s different about the equations in Example 2?

For these, you can solve by getting $x^2$ alone, and then take the _________________________________.
You can also get a binomial squared alone, and then take the _________________________________.
*DO NOT FORGET THE $+$ and $-$!

Example 2: Solve. If necessary, simplify all radicals \textit{and} give the radical rounded to 2 decimal places.

A) $5x^2 - 180 = 0$
B) $3x^2 = 24$

C) $4x^2 - 25 = 0$
D) $x^2 - \frac{1}{4} = 0$

E) $(3x + 4)^2 = 9$
F) $8(2x - 3)^2 = 16$

G) $9n^2 - 24n + 16 = 1$

Always make sure your radical answers are always simplified! Here’s a quick review …

A) $\sqrt{180}$
B) $\sqrt{252}$
C) $\sqrt{54p^3}$
D) $\frac{4}{\sqrt{81}}$
E) $\frac{7}{\sqrt{16}}$
Warm Up: Solve the equation $x^2 + 121 = 0$

This quadratic equation has NO real solutions. However, there are solutions existing outside the REAL number system. Let’s review that number system …

An **imaginary number** is any number whose square is -1. We use the letter $i$ to represent imaginary numbers.

Here’s the most important thing: $i^2 = -1$

That means that $i = i^3 = i^4 =$

Here’s a quick example: $\sqrt{-4} =$

*There is an *i* key on your calculator. Change your mode to $a+bi$. Sometimes it will help you. Always show your work to prove you did it by hand.*

**Example 1:** Simplify each radical completely.

A) $\sqrt{-5}$  
B) $\sqrt{-36}$  
C) $\sqrt{-12}$
So that’s imaginary numbers. When you combine a real number with an imaginary number, you get a **complex number**. A **complex number** is a real number plus an imaginary number. The standard form of complex numbers looks like this:

**NEVER leave $i$ to any power except 1 in your final answer. Simplify and combine like terms.**

**Example 2**: Write each expression as a complex number in standard form. $(a+bi)$
A) $(12-11i)+(-8+3i)$  
B) $(15-9i)-(24-9i)$  
C) $2(4-3i)-(10+6i)$

**Example 3**: Write each expression as a complex number in standard form. $(recall: i^2 = -1)$
A) $4(-6+i)$  
B) $(9-2i)(-4+7i)$  
*C) $(4-i)(4+i)$

*These are special complex number called CONJUGATES

**Example 4**: Write each expression as a complex number in standard form.
A) $(9-i)+(-6+7i)$  
B) $-4-(1+i)+(2+3i)$  
C) $(2-7i)(2+7i)$

D) $-5i(8-9i)$  
E) $(-8+2i)(4-7i)$  
F) $(3+4i)^2$

Let’s return to the equation from the beginning of class: $x^2 + 121 = 0$. You are now prepared to solve equations to find all solutions, whether they are REAL or COMPLEX (imaginary)

**Example 5**: Solve. Give all real and complex solutions.
A) $4x^2 + 72 = 0$  
B) $4x^2 + 23 = 3$  
C) $2x^2 + 44 = 4$
Completing the Square

After this lesson and practice, I will be able to ...

☐ solve by completing the square. (LT 10)

Do you remember perfect square trinomials? We learned how to factor them earlier in the unit. Let’s use that to help us solve an equation!

Example 1: Solve by finding square roots

\[ x^2 - 8x + 16 = 25 \]
\[ (x - 4)^2 = 25 \]

That’s a pretty great way to solve quadratic equations, but unfortunately, we won’t always have nice perfect square trinomials in our equations. Luckily for us, there’s a way to create perfect square trinomials. It’s called completing the square ...

Set up the following scenarios with your papers and fill in the table accordingly.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Number of 1-tiles needed to complete the square</th>
<th>Expression written as a square</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x^2 + 2x + _ )</td>
<td></td>
<td>((x + _)^2)</td>
</tr>
<tr>
<td>( x^2 + 4x + _ )</td>
<td></td>
<td>((x + _)^2)</td>
</tr>
<tr>
<td>( x^2 + 6x + _ )</td>
<td></td>
<td>((x + _)^2)</td>
</tr>
<tr>
<td>( x^2 + 8x + _ )</td>
<td></td>
<td>((x + _)^2)</td>
</tr>
<tr>
<td>( x^2 + 10x + _ )</td>
<td></td>
<td>((x + _)^2)</td>
</tr>
</tbody>
</table>

Answer these questions with your group. Consider the general statement \( x^2 + bx + c = (x + d)^2 \).

a. How is \( d \) related to \( b \)?

b. How is \( c \) related to \( d \)?

c. How can you obtain the numbers in the second column of the table directly from the coefficients of \( x \) in the expressions from the first column?
Let’s practice completing the square before we solve …

**Example 2:** Find the value of c that makes \( x^2 + bx + c \) a perfect square trinomial. Then write the expression as the square of a binomial.

A) \( x^2 + 16x + c \) \[\text{B) } x^2 + 22x + c \] \[\text{C) } x^2 - 9x + c \]

D) \( x^2 + 6x + c \) \[\text{E) } x^2 - 15x + c \]

---

**Solving quadratic equations of the form** \( x^2 + bx + c = 0 \) **by completing the square**

Steps to solving a quadratic equation by COMPLETING THE SQUARE:

1) Make sure the coefficient of \( x^2 \) is 1. (If it isn’t…divide by a suitable number!)

2) Put the constant on the right side and \( x^2 + bx \) on the left side.

3) Take \( \frac{1}{2} \) the coefficient of \( x \), square it, and add it to BOTH sides!!

4) Factor & solve using the square root method!

**Example 3:** Solve by completing the square: (If b is odd, use fractions not decimals)

A) \( x^2 + 12x = 28 \) \[\text{B) } x^2 - 6x + 5 = 0 \] \[\text{C) } x^2 - 9x + 20 = 0 \]

\[ x^2 + 12x + \boxed{\phantom{12}} = 28 + \boxed{\phantom{28}} \]
D) \(x^2 - 12x + 4 = 0\)  
E) \(x^2 - 8x + 36 = 0\)

Write original equation.
Write left side in the form \(x^2 + bx\).
Complete the square.
Write left side as a binomial squared.
Take square roots of each side.
Solve for \(x\).
Simplify.

**Example 4**: Solve each equation by completing the square.

A) \(x^2 - 10x + 8 = 0\)

B) \(x^2 + 4x - 4 = 0\)

C) \(x^2 + 4x + 1 = 0\)

D) \(x^2 + 6x + 12 = 0\)

E) \(x^2 - 3x - 5 = 0\)
Solving quadratic equations of the form \( ax^2 + bx + c = 0, a \neq 1 \) by completing the square.

Example 5: Solve by completing the square:

A) \( 2x^2 + 8x + 14 = 0 \)  
B) \( 3x^2 - 36x + 150 = 0 \)

C) \( 2x^2 + x = 6 \)  
D) \( x^2 + 21x = -98 \)

E) \( 3x^2 - 7x = 6 \)
The Quadratic Formula

Date: ___________ 

After this lesson and practice, I will be able to ...

☐ solve using the quadratic formula (with rationalized denominators). (LT 11)
☐ use the discriminant to determine the number and type of solutions. (LT 12)
☐ write quadratic questions given the real solutions. (LT 13)

So far, you have learned three strategies for solving quadratic equations … they are:

1. 

2. 

3. 

Each strategy has its own benefit depending on the characteristics of the equation. Today you’re going to see the fourth and final way of solving quadratic equations. It’s one of the most famous formulas in mathematics and it works on **ALL** quadratic equations!

Let’s first derive the Quadratic Formula using completing the square:

\[ ax^2 + bx + c = 0 \]  subtract c

\[ ax^2 + bx = -c \]  divide by a

\[ x^2 + \frac{b}{a}x + ____ = \frac{-c}{a} + ____ \]
The Quadratic Formula: For any quadratic equation \( ax^2 + bx + c = 0 \), the solutions are...

\[ x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \quad \text{or} \quad x = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \]

sometimes written as one solution with \( \pm \):

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

**Example 1:** Solve each equation. Write your solutions as exact values in simplest form (no decimals)

A) \( 3x^2 - 5x = 2 \) 

B) \( 2x^2 + 8x = -12 \) 

C) \( 9x^2 + 12x + 4 = 0 \)

D) \( 4x^2 - 8x + 1 = 0 \) 

E) \( 2x^2 = 7x - 8 \)
As we’ve seen, quadratic equations can have __________ or ______________ solutions. Let’s discover how to quickly determine which type of solution a given quadratic equation has …

**Discriminant:** Given \( ax^2 + bx + c = 0 \), the discriminant is ________________.

The discriminant does not include the radical symbol!

Fill out the table below.

<table>
<thead>
<tr>
<th>Equation</th>
<th>Value of Discriminant ( b^2 - 4ac )</th>
<th>Number (0, 1 or 2) and Type of Solutions (Real or Complex)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A) ( 3x^2 - 5x = 2 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>B) ( 2x^2 + 8x = -12 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C) ( 9x^2 + 12x + 4 = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>D) ( 4x^2 - 8x + 1 = 0 )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E) ( 2x^2 = 7x - 8 )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Summary:**

<table>
<thead>
<tr>
<th>Value of Discriminant</th>
<th>Number and Types of Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positive</td>
<td></td>
</tr>
<tr>
<td>Zero</td>
<td></td>
</tr>
<tr>
<td>Negative</td>
<td></td>
</tr>
</tbody>
</table>
**Example 2**: Find the discriminant and give the number and type of solutions. Do not solve.

A) \(x^2 + 10x + 23 = 0\)  
B) \(x^2 + 10x + 25 = 0\)  
C) \(x^2 + 10x + 27 = 0\)

- 0 real, 2 complex  
- 1 real  
- 2 real

**One more use for the discriminant!**

If the discriminant is a **perfect square**, the quadratic expression is **factorable** under rational numbers.

If it is **not** a perfect square, the quadratic expression is **prime** under rational numbers.

We now know how to find the solutions of a quadratic equation. Next we will do the reverse: **Find the quadratic equation given the solutions.**

We will use this in the next unit.

**Factored Form** – ________________, given \(r_1\) and \(r_2\) are solutions.

**Example 1**: Write a quadratic equation in factored form with the following solutions:

A) 2 and -4  
B) 4  
C) \(-\frac{1}{2}\) and -7

**Example 2**: Write a quadratic equation in factored form with the following solutions.

A) -1 and 3  
B) -4 and 0  
C) -10

**Example 3**: Write a quadratic equation in **STANDARD** form with the following solutions.

A) -2 and 2  
B) -5  
C) -5 and \(\frac{1}{4}\)