Math League SCASD

2019-20

Meet #1

Algebra

Self-study Packet

Problem Categories for this Meet:

1. Mystery: Problem solving
2. Geometry: Angle measures in plane figures including supplements and complements
3. Number Theory: Divisibility rules, factors, primes, composites
4. Arithmetic: Order of operations; mean, median, mode; rounding; statistics
5. Algebra: Simplifying and evaluating expressions; solving equations with 1 unknown including identities
Important information you need to know about Algebra...

Simplifying and evaluating expressions; solving equations with 1 unknown including identities

Equations (The Basics)

- Goal: Get the Variable Alone
- Whatever you do to one side of the equation, you must do to the other!
- Undo equations using the reverse order of operations.
- Use inverse operations (opposites) to get the variable alone.
  - Multiplication and Division undo each other.
  - Adding and Subtracting undo each other.
  - Squaring and Taking the Square Root of a number undo each other.

Simplifying Expressions

Coefficient: a number multiplying a variable. The coefficient of $7a$ is 7. The coefficient of $-2xy$ is $-2$. The coefficient of $x$ is 1, because 1 multiplied by $x$ is $x$.

Like terms: have exactly the same variable(s) raised to the same power

<table>
<thead>
<tr>
<th>Like Terms</th>
<th>Not Like Terms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7x$ and $9x$</td>
<td>$3x$ and $3y$</td>
</tr>
<tr>
<td>$4x^2$ and $-12x^2$</td>
<td>$3x^3$ and $4x^2$</td>
</tr>
<tr>
<td>$-ab^3c^2$ and $4ab^3c^2$</td>
<td>$5a^4b$ and $12ab^4$</td>
</tr>
</tbody>
</table>

To combine like terms, add their coefficients. Keep the variable and the exponent the same!

Distributive Property: To distribute, multiply what is directly outside of the parentheses by everything inside the parentheses.
For example: $a(b + c) = ab + ac$
$7(x - 5) = 7x - 35$

Identity: an equation that is always true.

For example $7x + 5 = 7x + 2 + 3$ is an identity, because no matter what value you substitute for $x$, the equation will always be true.
1) If $A = 11$, and $B = -4$, and $C = 5$, then what is the value of $3A - 7B + 8C$?

2) What value of $D$ makes the following equation true?

$$17 - 3D = 5(1 - D) - 3(4 - 2D)$$

3) If the following expression simplifies to $Ax + By$, then what is the value of $10A - 9B$?

$$5(4x + y) - 7(2x + 3y) - 6(x - 8y)$$

**Answers**

1) 

2) 

3) 

Solutions to Category 5  
Algebra  
Meet #1 - October, 2017

1) For simplicity, substituting $X = 11$, $Y = -4$, and $Z = 5$, then $3Z - 7Y + 8X = 3(5) - 7(-4) + 8(11) = 15 + 28 + 88 = 131.$  

2) Again, for simplicity, substituting $X$ for the ghost:

\[
17 - 3X = 5(1 - X) - 3(4 - 2X)
\]
\[
17 - 3X = 5 - 5X - 12 + 6X
\]
\[
17 - 3X = -7 + X
\]
\[
24 = 4X
\]
\[
6 = X
\]

3) $5(4x + y) - 7(2x + 3y) - 6(x - 8y) = 20x + 5y - 14x - 21y - 6x + 48y = 0x + 32y$

So, $A = 0$ and $B = 32$. Then $10A - 9B = 10(0) - 9(32) = 0 - 288 = -288$. 

\textbf{Answers}

1) 131  
2) 6  
3) -288
Category 5
Algebra
Meet #1 - October, 2015

1) Find the value of $5x^2 - 2x^3$ if $X = -4$.

2) For what value of $N$ is $2N - 6[3N - 2(7N - 4)] - 5N = 456$?

3) If Jean were ten years older, she would be three years older than she would have been if she were four times as old as she was 38 years ago. How many years old will Jean be in 27 years?

Answers
1) _____
2) _____
3) _____

Henry Ford introduced the Model T on October 2, 1908.
Solutions to Category 5
Algebra
Meet #1 - October, 2015

1) \[5x^2 - 2x^3\]
   \[= 5(-4)^2 - 2(-4)^3\]
   \[= 5(16) - 2(-64)\]
   \[= 80 + 128\]
   \[= 208\]

2) \[2N - 6[3N - 2(7N - 4)] - 5N = 456\]
   \[2N - 6[3N - 14N + 8] - 5N = 456\]
   \[2N - 6[-11N + 8] - 5N = 456\]
   \[2N + 66N - 48 - 5N = 456\]
   \[63N - 48 = 456\]
   \[63N = 504\]
   \[N = 8\]

3) Let \(X\) = Jean's age now, in years
   \(X + 10\) = Jean's age 10 years from now, in years
   \(X - 38\) = Jean's age 38 years ago, in years

   \[X + 10 = 4(X - 38) + 3\]
   \[X + 10 = 4X - 152 + 3\]
   \[X + 10 = 4X - 149\]
   \[-3X = -159\]
   \[X = 53\]
   \[X + 27 = 80\]

So, Jean is now 53 years old. In 27 years, she will be 53 + 27, or 80 years old.
Category 5
Algebra
Meet #1 - October, 2013

1) If \( P = -1 \) and \( R = \frac{1}{2} \) then evaluate \(-P^2 + 4R^4 - 6PR\).
Express your answer as a decimal.

2) Simplify: \( 4(2X - 5) + 3(7X - 6) - 2(5X-1) \)

3) Situation #1: \( 10A + 1 = 7(2A - 5) \)
Situation #2: \( 3C^2 = 75 \)
Situation #3: The following equation is an identity in \( N \):
\( 4N - 7 + W = -2(3N + 4) + 10N \)
Finally, find the value of \( X \) if
\( X = \) (the smaller value of \( C \))(the larger value of \( C \)) - \( AW \)

Answers
1) 
2) 
3) 

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Solutions to Category 5
Algebra
Meet #1 - October, 2013

<table>
<thead>
<tr>
<th>Answers</th>
<th>1) If ( P = \frac{1}{2} ) and ( R = \frac{1}{2} ) then (-P^2 + 4R^4 - 6PR)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 2.25</td>
<td>(\nabla) (-(-1)^2 + 4\left(\frac{1}{2}\right)^4 - 6(-1)\left(\frac{1}{2}\right))</td>
</tr>
<tr>
<td>2) 19X - 36</td>
<td>(-1 + 4\left(\frac{1}{16}\right) + 3)</td>
</tr>
<tr>
<td>3) - 16</td>
<td>(-1 + \frac{1}{4} + 3)</td>
</tr>
<tr>
<td></td>
<td>(= 2.25)</td>
</tr>
</tbody>
</table>

2) Simplify:
\[
4(2X - 5) + 3(7X - 6) - 2(5X-1)
= 8X - 20 + 21X - 18 - 10X + 2
= 19X - 36
\]

3) Situation #1:
\[
10A + 1 = 7(2A - 5)
10A + 1 = 14A - 35
36 = 4A
9 = A
\]

Situation #2:
\[
3C^2 = 75
C^2 = 25
C = \pm 5
\]

Situation #3: The following equation is an identity:
\[
4N - 7 + W = -2(3N + 4) + 10N
4N - 7 + W = -6N - 8 + 10N
10N + W = -1 + 10N
W = -1
\]
\[
X = (the \ smaller \ value \ of \ C)(the \ larger \ value \ of \ C) - AW
= (-5)(5) - (9)(-1)
= -25 + 9
= -16
\]
Category 5 – Algebra

Express your answers as common fractions (not decimals) if necessary.

1. Find the value of $x$ that makes the following equality true:

\[
\frac{x + 2}{2} + \frac{x - 2}{3} = \frac{x}{2}
\]

2. Find the value of $M$ that will make the equation below an identity:

\[
M \cdot x + 2 \cdot M \cdot (x + 1) + 3 \cdot M \cdot (x + 2) = 3 \cdot x + 4
\]

3. We define the operation $\clubsuit$ as follows: $A \clubsuit B \equiv A^2 - B$

Find the value of a natural number $N$ for which: $8 \clubsuit (N \clubsuit 4) = 7 \clubsuit 17$

**Answers**

1. 

2. 

3. 

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Solutions to Category 5 - Algebra

1. Multiplying the equation by the common denominator, 6, we get:
   \[ 3 \cdot (x + 2) + 2 \cdot (x - 2) = 3 \cdot x \text{, or -} \]
   \[ 3 \cdot x + 6 + 2 \cdot x - 4 = 3x \text{, which we can rearrange into -} \]
   \[ 2 \cdot x = 4 - 6 = -2 \text{, or } x = -1 \]

\[
\text{Answers} \\
1. \ -1 \\
2. \ \frac{1}{2} \\
3. \ 6 \\
\]

2. If we simplify the left side of the equation:
   \[ M \cdot x + 2 \cdot M \cdot (x + 1) + 3 \cdot M \cdot (x + 2) = \]
   \[ M \cdot x + 2 \cdot M \cdot x + 2 \cdot M + 3 \cdot M \cdot x + 6 \cdot M = \]
   \[ 6 \cdot M \cdot x + 8 \cdot M = 2 \cdot M \cdot (3 \cdot x + 4) \text{ and this has to equal the right side of the equation, which is } 3 \cdot x + 4 \text{, so } 2 \cdot M = 1, \text{ or } M = \frac{1}{2} \]

3. \[ 8\spadesuit(N\spadesuit4) = 8^2 - (N\spadesuit4) = 64 - (N^2 - 4) = 68 - N^2 = 7\spadesuit17 = 7^2 - 17 = 32 \]
   So we’re left with: \[ 68 - N^2 = 32 \text{, or } N^2 = 36 \text{, and since } N \text{ is a natural number, the solution is } N = 6 \]
Category 5 - Algebra
Meet #1, October 2009

1. Given these values:
   \[ A = \frac{1}{3}, B = 5, C = 1, D = \frac{2}{3}, E = \frac{3}{2}, F = -1 \]
   What is the value of the expression: \( A \cdot \frac{B+C}{D\cdot E} + F \) ?

2. Solve for \( x \): (What is the value of \( x \) that makes this true?)
   \[ 3 \cdot (x - 5) + 40 = (-2) \cdot x + 5 \]

3. For what value of \( M \) will the solution of the following equation be \( x = \frac{M}{2} \)?
   \[ 2 \cdot M + x = 3 \cdot (M - 1) \]

Answers
1. _______________
2. _______________
3. _______________
Solutions to Category 5
Algebra
Meet #1, October 2009

1. \(A \cdot \frac{B+C}{D\cdot E} + F = \frac{1}{3} \cdot \frac{5+1}{\frac{2}{3} \cdot \frac{3}{2}} + (-1) = \frac{1}{3} \cdot \frac{6}{1} - 1 = 2 - 1 = 1\)

2. \(3 \cdot (x - 5) + 40 = (-2) \cdot x + 5\)
   \[3x - 15 + 40 = -2x + 5\]
   \[5x + 25 = 5\]
   \[5x = -20\]
   \[x = -4\]

3. If we know that \(x = \frac{M}{2}\) is the solution, we just have to put it back in the original equation and solve for M:
   \[2M + x = 2M + \frac{M}{2} = 3M - 3\] which is simplified to \(\frac{M}{2} = 3\) or \(M = 6\).

   We could have also simplified the original equation to be \(x = M - 3\) and then sub in \(x = \frac{M}{2}\) to get the same result.

Answers

1. 1
2. -4
3. 6
1. Find the value of M so that the equation below is an identity.

\[ 3(2x + 4M) + 16 + 5x = 8(3x - 4) - 13x \]

2. Evaluate the expression below for \( x = 4 \) and \( y = -\frac{1}{2} \).

\[ (2x + 8y) - \left( 6y - \frac{x}{2} \right) \]

3. Solve for \( x \):

\[ 3x + 7 + 2(8 + 4x) = 5(2x + 9) - x \]
Solutions to Category 5
Algebra
Meet #1, October 2007

Answers

1. 
\[ 3(2x + 4M) + 16 + 5x = 8(3x - 4) - 13x \]
\[ 6x + 12M + 16 + 5x = 24x - 32 - 13x \]
\[ 11x + 12M + 16 = 11x - 32 \]
\[ 12M = -48 \]
\[ M = -4 \]

2. 
\( (2x + 8y) - \left( 6y - \frac{x}{2} \right) = \)
\[ \left( 2(4) + 8 \left( -\frac{1}{2} \right) \right) - \left( 6 \left( -\frac{1}{2} \right) - \frac{4}{2} \right) = \]
\[ (8 - 4) - (-3 - 2) = \]
\[ 4 - (-5) = \]
\[ 9 \]

3. 
\[ 3x + 7 + 2(8 + 4x) = 5(2x + 9) - x \]
\[ 3x + 7 + 16 + 8x = 10x + 45 - x \]
\[ 11x + 23 = 9x + 45 \]
\[ 2x = 22 \]
\[ x = 11 \]