Problem Categories for this Meet (in addition to topics of earlier meets):

1. Mystery: Problem solving
2. Geometry: Area and perimeter of polygons
3. Number Theory: Divisibility GCF, LCM, prime factorization
4. Arithmetic: Fractions, terminating and repeating decimals, percents
5. Algebra: Word problems with 1 unknown; working with formulas; reasoning in number sentences
Meet #2 – Arithmetic

**Ideas you should know:**

**Multiplying fractions:**

\[
\frac{2}{3} \times \frac{5}{7} = \frac{2 \times 5}{3 \times 7} = \frac{10}{21}
\]

\[
\frac{2}{3} \times \frac{3}{7} = \frac{2}{21}
\]

Cancel before x for speed

**Dividing Fractions:**

\[
\frac{2}{3} \div \frac{1}{6} = \frac{2 \times 6}{3 \times 1} = \frac{12}{3} = 4
\]

\[
A \div \frac{B}{C} = A \times \frac{C}{B}
\]

**Reciprocal:** Multiplicative Inverse.

Reciprocal of 3 = 1/3. Reciprocal of 2/7 = 7/2.
Divide by Y = Multiply by reciprocal of Y

**Writing fractions of fractions**

Cancel before x for speed

\[
\frac{2}{3} \div \frac{1}{6} = \frac{2 \times 6}{3 \times 1} = \frac{12}{3} = 4
\]

**Adding Fractions – common denominator**

\[
\frac{1}{2} + \frac{1}{6} = \frac{3}{6} \times \frac{1}{2} = \frac{3}{6} + \frac{1}{6} = \frac{4}{6} = \frac{2}{3}
\]

\[
\frac{1}{7} + \frac{1}{5} = \frac{5}{35} + \frac{7}{35} = \frac{12}{35}
\]
What do you mean by “of”?

\[
\frac{2}{3} \text{ of } 24 = \frac{2}{3} \times 24 = \frac{2}{3} \times \frac{24}{1} = \frac{48}{3} = 16
\]

Of = Times

Fraction in Lowest Terms

\[
\frac{24}{32} = \frac{3 \times 8}{4 \times 8} = \frac{3}{4}
\]

Repeating decimal equivalent:

\[
\frac{1}{3} = 0.333333333... = 0\bar{3}
\]

\[
\frac{1}{9} = 0.11111... = 0\bar{1}
\]

\[
\frac{2}{9} = 0.22222... = 0\bar{2}
\]

\[
\frac{1}{99} = 0.01010101... = 0\bar{0}1
\]

\[
\frac{2}{99} = 0.02020202... = 0\bar{0}2
\]

\[
\frac{1}{999} = 0.001
\]

\[
\frac{1}{999} = 0.001
\]

\[
103/999 = \bar{0.103}
\]

0.17171717... = \bar{0.17}

\[
0.51111... = 0.\overline{51} = ?
\]

1/9 = 0.11111... 1/90 = 0.011111...

0.511111... = 1/2 + 1/90 = 45/90 + 1/90 = 46/90 = 23/45

Improper Fraction \[
\frac{10}{3} \quad \frac{3}{2}
\]

Mixed Numeral \[
2!
\]

≡ Mike is 50% taller than Bob: This means he’s 1.5 times as tall, not ! as tall!

“I ate 50% as much as you” = half as much.
“I ate 50% more than you” = 1.5 times as much
“I ate 100% as much as you” = same
“I ate 100% more than you” = twice as much
“I ate 200% more than you” = 3 times as much
“I ate 50% of you” = well, nevermind.

≡ The price is 1/3 higher: The price is 1+1/3 as high. If the original price was $30, then 1/3 higher means it’s $40.
“What fraction is this repeating decimal?”

Another way to figure it out:

\[ X = 0.\overline{12} \quad times \quad 100: \]
\[ 100X = 12.\overline{12} \quad subtract \ X: \]
\[ 99X = 12 \quad \therefore X = 12/99 = 4/33 \]

If digits before the repeating pattern:

\[ X = 0.1\overline{23} \]
\[ 10X = 1.2\overline{3} \]
\[ 1000X = 123.\overline{23} \quad subtract: \]
\[ 990X = 122 \quad so, X = \frac{122}{990} = \frac{61}{495} \]

“15th digit in the decimal expansion of” problems

What is the 15th digit of the decimal expansion of 1/7? 1/7 = 0.142857 You could just write it out and count digits. Another way is to say digit 3 is 2, and every 6th digit after that is also a 2, and 15=3+6x2, so it’s also 2.

What is the 601st digit of the decimal expansion of 2/7= 0.285714?
Answer: It’s 600 digits past the 1st, so it’s the same as the 1st, or 2.

“What is 2/3 of 25% of 3/7 of 4/9 of 81” problems

These are simply multiplication – with a lot of cancellation usually.

\[ 2 \cdot \frac{25}{100} \cdot \frac{3}{7} \cdot \frac{4}{9} \cdot 81 \]
Rewrite 25/100 as 1/4, cancel 3’s and 4’s:

\[ \frac{2 \cdot 1}{3 \cdot 4 \cdot 7 \cdot 9} \cdot \frac{3}{100} \cdot \frac{4}{7} \cdot \frac{9}{81} \]
\[ or \quad \frac{2 \cdot 1 \cdot 3}{1 \cdot 1 \cdot 7 \cdot 9} \cdot \frac{3}{4 \cdot 7} \cdot \frac{9}{81} \]
and also cancel 9s from 1/9 and 81, and so we get 2x9/7 or 18/7 or 2 4/7.

Adding or subtracting repeating decimals

If you have 0.33333... plus 0.11111... you get 0.44444... which makes sense if you look at them as fractions: 3/9 + 1/9 = 4/9. It’s tricky if the two repeating patterns have a different length:

From the 1999 meet: What is 0.\overline{51} + 0.\overline{2}? Answer: Write 0.2... as 0.22... and then it’s 51/99+22/99 = 73/99 or 0.737373...

Dividing repeating decimals

This seems harder, but you can often do it in your head using fractions:

What is 0.\overline{21} + 0.\overline{60}? Answer: \[
\frac{21}{99} + \frac{60}{99} = \frac{21}{99} \times \frac{99}{60} = \frac{21}{60} = \frac{7}{20} \cdot \frac{3}{20} = \frac{7}{20}
\]
1) Express the following sum as an improper fraction in lowest terms:

\[
\frac{1}{2} + 0.75 + 2\frac{1}{4}
\]

2) Express this sum as a fraction in lowest terms:

\[
0.2 + 0.22 + 0.2\%
\]

3) When the repeating decimal \(0.436363636\ldots\) is written as a common fraction, the result is \(\frac{A}{B}\). What is the value of \(A + B\)?

**Answers**

1) _______

2) _______

3) _______
1) \[ \frac{1}{2} + 0.75 + 2 \frac{1}{4} = 0.5 + 0.75 + 2.25 = 3.5 = \frac{7}{2}. \]

2) \[ 0.2 + 0.22 + 0.2\% = 0.2 + 0.22 + 0.002 = 0.422 = \frac{422}{1000} = \frac{211}{500}. \]

3) Let \[ X = 0.4363636 \ldots \]

Then \[ 100X = 43.6363636 \ldots \]

\[ 100X - X = (43.6363636 \ldots) - (0.436363636 \ldots) \]

\[ 99X = 43.2 \]

\[ X = \frac{43.2}{99} = \frac{432}{990} = \frac{216}{495} = \frac{72}{165} \]

Finally, \[ 72 + 165 = 237. \]
1) I watched only 27 minutes of a three-hour baseball game. What percent of the game did I miss?

2) Abraham Lincoln was a tall man but he was 20% taller at 228 centimeters when he wore his famous "stovepipe" hat. How many centimeters tall was his hat?

3) What is $\frac{5}{4}$ of $2.5\%$ of $1.7$ of $0.24$ of 1440? Express your answer as a common fraction.

Answers
1) _____
2) _____
3) _____
Solutions to Category 4
Arithmetic
Meet #2 - November, 2015

1) I missed 180 - 27, or 153 minutes of the game.
   \[ \frac{153}{180} = 0.85 = 85\% \]

2) Let \( X \) = Abraham Lincoln's height in centimeters.
   \[ X + (20\% \ of \ X) = 228 \]
   \[ X + 0.2X = 228 \]
   \[ 1.2X = 228 \]
   \[ X = \frac{228}{1.2} \]
   \[ X = 190 \]
   \[ 228 - 190 = 38 \]
   So, Lincoln is 190 cm tall while his hat is 38 cm tall.

3) \[ \frac{5}{4} \ of \ 2.5\% \ of \ 1.7 \ of \ 0.24 \ of \ 1440 \]
   \[ = \frac{5}{4} \ of \ \frac{25}{1000} \ of \ \frac{16}{9} \ of \ \frac{11}{45} \ of \ 2 \times 2 \times 2 \times 3 \times 3 \times 5 \]

   Factoring numerators and denominators and "cancelling" common factors yields the fraction \( \frac{176}{9} \).
Category 4  
Arithmetic  
Meet #2 - November, 2013

1) The harmonic mean of two numbers is the reciprocal of the arithmetic mean (or average) of their reciprocals. It is one of the three Pythagorean means and is a better way to calculate certain types of averages, like the average of rates. Using this concept, find the harmonic mean of 40 mph and 60 mph.

2) At the vintage Filene's Basement in Downtown Boston, the following pricing policy was in effect in 1965:

   - new products: full price
   - products at least 7 days old: 25% off the original price
   - products at least 14 days old: 50% off the original price
   - products at least 21 days old: 75% off the original price
   - products at least 28 days old: donated to charity

If Martha bought a 9-day-old dress that was originally priced at $8.92 and a 25-day-old pair of shoes that was originally priced at $9.48, then how much money did she pay in all?

3) Simplify: \[ \frac{\frac{3}{4} + 2.625}{7.6 - \frac{5}{3}} \] Express your answer as a common fraction.

<table>
<thead>
<tr>
<th>ANSWERS</th>
</tr>
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<tbody>
<tr>
<td>1) _______ mph</td>
</tr>
<tr>
<td>2) $______</td>
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<tr>
<td>3) ________</td>
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</tbody>
</table>
Solutions to Category 4
Arithmetic
Meet #2 - November, 2013

Answers

1) 48
2) 9.06
3) \( \frac{9}{16} \)

\[
1) \quad \frac{1}{\frac{1}{2} \left( \frac{1}{40} + \frac{1}{60} \right)} - \frac{1}{\frac{1}{2} \left( \frac{60}{2400} + \frac{40}{2400} \right)} - \frac{1}{\frac{100}{2400}} = \frac{1}{48} = 48
\]

2) dress: 0.75 (8.92) = $6.69
   shoes: 0.25 (9.48) = $2.37
   total: $9.06

3) \[
\frac{\frac{3}{4} + 2.625}{\frac{7.6}{3} - \frac{5}{3}} = \frac{0.75 + 2.625}{\frac{2}{3} - \frac{5}{3}} = \frac{27}{8} = \frac{27 \times \frac{1}{6}}{\frac{48}{16}} = \frac{9}{16}
\]
Category 4 – Arithmetic

1. Express $\frac{1}{5} + \frac{1}{3}$ as a decimal.

   *Use bar notation where appropriate.*

2. Express $3.\overline{39}$ as a common fraction.

   *A common fraction is of the form $\frac{m}{n}$ where $m$ and $n$ share no common factors.*

3. In a certain town, 50% of the population are children, and 48% of children are girls.

   Half the boys, and a third of the girls, like to watch SpongeBob on TV.

   Assuming no adults do, what percentage of the population likes SpongeBob?

---

**Answers**

1. _______________
2. _______________
3. ____________ %
Solutions to Category 4 - Arithmetic

1. \(\frac{1}{5} + \frac{1}{3} = \frac{3+5}{15} = \frac{8}{15} = 0.5\overline{3}\)
   
   This can be verified with long division.

2. If we call \(x \equiv 3.\overline{39}\) then we can have \(100 \cdot x = 339.\overline{39}\), and subtracting one from the other we get \(99 \cdot x = 336\) or \(x = \frac{336}{99} = \frac{112}{33}\)

3. The overall percentage of boys is \(50\% \times 52\% = 26\%\), and the overall percentage of girls is \(50\% \times 48\% = 24\%\).
   
   Half the boys would be 13\% of the population, and a third of the girls would be 8\%, for a total of 21\% SpongeBob fans.

Answers

1. 0.5\overline{3}
2. \(\frac{112}{33}\)
3. 21\%
1. Express $\frac{5}{16} + \frac{1}{9}$ as a decimal.

   *Use bar notation where appropriate.*

2. Write $\frac{0.18}{1-0.6}$ as a simple fraction.

3. Mr. Bell spent a fifth of his money, then 12.5% of the remaining amount, then 10% of what was left, and finally a third of the remaining balance.

   If he initially had $200, how much is left at the end?

---

**Answers**

1. _______________
2. _______________
3. _______________
Solutions to Category 4 - Arithmetic
Meet #2, December 2009

1. \( \frac{5}{16} + \frac{1}{9} = 0.3125 + 0.\overline{1} = 0.4236\overline{1} \)

2. If you recall that \( \frac{1}{11} = 0.\overline{09} \) then you’d realize that \( \frac{2}{11} = 0.\overline{18} \), if not you can write
\[
x = 0.\overline{18}, 100x = 18.\overline{18}
\]
to get \( 99x = 18 \) or \( x = \frac{18}{99} = \frac{2}{11} \).

In the denominator we have \( 1 - 0.\overline{6} = 0.\overline{3} = \frac{1}{3} \) so overall we have \( \frac{\frac{2}{11}}{\frac{1}{3}} = \frac{6}{11} \).

3. We can follow this in steps:
\[
\begin{align*}
$200 \cdot \frac{4}{5} &= $160 \\
$160 \cdot 87.5\% &= $160 \cdot \frac{7}{8} = $140 \\
$140 \cdot 90\% &= $126 \\
$126 \cdot \frac{2}{3} &= $84
\end{align*}
\]
1. Bill spends 30% of $\frac{3}{8}$ of the $960 he has in the bank. How much will he have left in the bank after he spends this amount?

2. Simplify $\frac{.57}{.86}$ as a fraction in simplest terms.

3. When the fractions $\frac{7}{12}$ and $\frac{5}{7}$ are added and converted to decimal form, the decimal will have a six digit repetition in it. What is that 6 digit repetition? (note: write your answer as a 6 digit number without decimals or repeating bars. For example, if the decimal was $.978123456$ you would write your answer as 123456.)

Answers

1. _______________
2. _______________
3. _______________
Solutions to Category 4
Arithmetic
Meet #2, November 2007

Answers

1. \[30\% \text{ of } \frac{3}{8} \text{ of } \$960 = 30\% \text{ of } \frac{3}{8} \times \frac{960}{1} = 30\% \text{ of } 360 = .3(360) = \$108\]
   If he spends $108 he has $960 – $108 = $852 left over.

2. \[\frac{2}{3}\]

2. a) There are several ways to convert repeating decimals into fractions. Once you know the process, this is the fastest:
\[
\frac{.5\overline{7}}{.86\overline{8}} = \frac{57 - 5}{100 - 10} = \frac{52}{90} = \frac{52}{78} = \frac{4}{9} = \frac{2}{6} = \frac{1}{3}
\]

b) You could also look at \(0.\overline{57}\) as being \(5.7/10 = 5\frac{7}{9} = 52/9 \times 1/10 = 52/90\)
   and do the same with the other decimal.

\[x = 0.\overline{57}\]
\[100x = 57.7\]
\[-10x = 5.7\]
\[90x = 52\]
\[x = \frac{52}{90}\]

\[\frac{7}{12} = 0.5833333333333333\]

3. \[\frac{5}{7} = 0.71428571428571428571\]

You could also add the fractions: \[\frac{7}{12} + \frac{5}{7} = \frac{109}{84} = 1\frac{25}{84}\] and then divide 25 by 84.
Category 4
Arithmetic
Meet #2, December 2005

1. Martin is supposed to get 20% of \( \frac{5}{7} \) of 0.35 of the $2940 in profits his company made last week. What is the dollar amount that Martin should get?

2. What is the positive difference between \( \frac{2}{3} \) and \( \frac{2}{3} \)? Express your answer as a mixed number in lowest terms.

3. What is the 53rd digit to the right of the decimal point in the decimal expansion of \( \frac{9}{37} \)?

Answers
1. _______________
2. _______________
3. _______________
Solutions to Category 4
Arithmetic
Meet #2, December 2005

Answers

1. Each use of the word “of” in the statement of the problem translates to a multiplication that must be done. Let’s convert to fractions as follows: \( \frac{20}{100} \times \frac{5}{7} \times \frac{35}{100} \times \frac{2940}{20} \). Now let’s simplify and see what we get:

\[
\frac{1}{5} \times \frac{5}{7} \times \frac{2940}{20} = \frac{1}{2940} = \frac{1}{2}
\]

2. First let’s find the value of each fraction.

\[
\frac{2}{3} \div \frac{4}{1} = \frac{2}{3} \cdot \frac{1}{4} = \frac{2}{12} = \frac{1}{6} \quad \text{and} \quad \frac{2}{3} \div \frac{1}{4} = \frac{2}{1} \cdot \frac{4}{3} = \frac{8}{3}
\]

The positive difference between these two fractions is

\[
\frac{8}{3} - \frac{1}{6} = \frac{32}{12} - \frac{2}{12} = \frac{30}{12} = 2 \frac{6}{12} = 2 \frac{1}{2}
\]

3. We have to divide 9 by 37, as shown at left, to find the decimal equivalent of \( \frac{9}{37} \). When we get a remainder of 9, the three-digit pattern repeats. Using bar notation, we write the decimal equivalent for \( \frac{9}{37} \) as \( 0.243 \). Since 54 = 18 \times 3, the 54th digit to the right of the decimal point will be the last digit in the repeating pattern, which is a 3. The 53rd digit must be the second-to-last digit in the pattern, which is a 4.