Math League SCASD

2022-23

Meet #2

Algebra

Self-study Packet

Problem Categories for this Meet (in addition to topics of earlier meets):

1. Mystery: Problem solving
2. Geometry: Area and perimeter of polygons
3. Number Theory: Divisibility GCF, LCM, prime factorization
4. Arithmetic: Fractions, terminating and repeating decimals, percents
5. **Algebra: Word problems with 1 unknown; working with formulas; reasoning in number sentences**

Meet #2 – Algebra

Ideas you should know:

≡ Common Fraction: $\frac{2}{3}$, $\frac{9}{2}$ ~~$Not: \frac{10}{6}$~~ ~~$Not: 2\frac{1}{6}$~~

≡ Money Answers: "What is One-Quarter of a dollar, minus two cents?"

23¢ $0.23 $.23 ~~Not 0.23¢~~ ~~Not 23~~

≡ Square Root of a Product: "What is the square root of 14x21 x 6?"

$$\sqrt{(14 \cdot 21) \cdot (6)} = ?$$

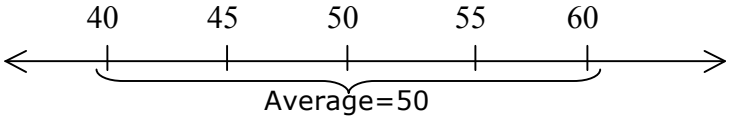
☹ Slow hard way – "Multiply it out first":

14x21=294, 294x6=1764, $\sqrt{1764} = ?$ "Um, Are we allowed to use calculators? How are we supposed to do this?"

☺ Faster way – "Factor it first":

From question:	14	21	6
Factor:	2 x 7	3 x 7	2 x 3
Regroup factors:	2 x 2	3 x 3	7 x 7
Now do !	$\sqrt{2 \times 2} = 2$	=3	=7
Answer:		2x3x7=42	

≡ "Five consecutive multiples of 5 have a sum of 250 …" problems:



If 5 numbers add to 250, the average is 50. That's also the middle number, so the five numbers are 40,45,50,55,60. Often the problem will ask for the 2nd number times the 4th number (45 x 55 here).

≡ "Four times the sum of a number and one is two more than seven times an amount one less than the number. Find the number."

So confusing!

Make N be the number you don't know, then translate the words to Algebra:

Four times the sum of a number and one is two more than English

4 x (N + 1) - 2 = Algebra

Seven times an amount one less than the number English

7 x (N - 1) Algebra

Or: 4 (N+1) – 2 = 7 (N-1)

Find the number. English

Solve for N. Algebra

First, distribute, then combine like terms:

4N + 4 – 2 = 7N – 7 (Add 7 – 4N to each side)

4 – 2 + 7 = 7N – 4N

9 = 3N

3 = N

Then check this answer in the original problem!

"Four times (3+1) is two more than 7(3-1)" Yes, 14=14

Category 5
Algebra
Meet #2 - December, 2020

1) A rectangle and an equilateral triangle have the same perimeter. One side of the triangle measures 24 inches. The length of the rectangle is three times as long as its width. How many inches long is the rectangle?

2) If A + 2B = 17 and C + 3D = 18, then what is the value of 4A + 8B - (3C + 9D) - 12 + 8 ?

3) Mathematics and science and loaded with fun and useful formulas to help discern information from given data. The formula that relates a temperature in degrees Fahrenheit, F, to degrees Celsius, C, is

$$F = \frac{9}{5}C + 32$$

The record high temperature for Boston in the month of December occurred on December 7, 1998 and was 76 degrees Fahrenheit. The record low occurred on December 29, 1933 and was 17 degrees below zero, Fahrenheit. What is the positive difference, in degrees Celsius, between the record high and low temperatures for December? Round your answer to the nearest whole number.

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #2 - December, 2020

Answers	
1)	27
2)	10
3)	52

1) The perimeter of the equilateral triangle is (3)(24), or 72 inches. If the perimeter of the rectangle is the same as that of the triangle, then half its perimeter is 36 inches, or the sum of its length and width. If the length is three times the width, then W + 3W = 36, 4W = 36, and W = 9 and the length is 27 inches.

2) If A + 2B = 17, then multiplying both sides by 4 yields 4A + 8B = 68.
If C + 3D = 18, then multiplying both sides by 3 yields 3C + 9D = 54.
Then 4A + 8B - (3C + 9D) - 12 + 8
= 68 - (54) - 12 + 8
= 14 - 12 + 8
= 2 + 8
= 10.

3) The straightforward method is to convert the two Fahrenheit temperatures to Celsius and then subtract and round:
record high: F = 9/5 C + 32, 76 = 9/5 C + 32, 44 = 9/5 C, C = 220 / 9.
record low: F = 9/5 C + 32, - 17 = 9/5 C + 32, - 49 = 9/5 C, C = - 245 / 9.
The difference is 220 / 9 - (- 245 / 9) = 465 / 9 = 51.666... which, rounded to the nearest whole number, is 52 degrees Celcius.

Category 5
Algebra
Meet #2 - December, 2018

1) **There are three positive integers, such that one is four more than another, while the third is twice the sum of the other two. The sum of all three integers is 54. What is the largest of the three integers?**

2) **If 2E + 3H = 21 and 4M - J = - 31 then what is the value of 8E + 12H - (8M - 2J) + 7 ?**

3) $F=\frac{9}{5}C+32$ **is the formula that converts a temperature in Celcius (C) degrees to Fahrenheit (F) degrees. The Massachusetts town of Concord set a record high temperature this past October with a temperature of 95 degrees F. What was that temperature in Celcius?**

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #2 - December, 2018

Answers
1) 36
2) 153
3) 35

1) **Let** X = the smallest integer.
$X + 4$ = the next larger integer
$2(X + X + 4)$ = the third integer

Then $(X) + (X + 4) + 2(X + X + 4) = 54$
$2X + 4 + 2X + 2X + 8 = 54$
$6X + 12 = 54$
$6X = 42$
$X = 7$
$7 + 11 = 18$
$2(7 + 11) = 36.$

Therefore, the largest of the three integers is 36.

2) **If $2E + 3H = 21$, then $8E + 12H = 4(2E + 3H) = 4(21) = 84.$**
If $4M - J = -31$, then $8M - 2J = 2(4M - J) = 2(-31) = -62.$
Then $8E + 12H - (8M - 2J) + 7 = 84 - (-62) + 7 = 84 + 62 + 7 = 153.$

3)

$$F = \frac{9}{5}C + 32$$

$$95 = \frac{9}{5}C + 32$$

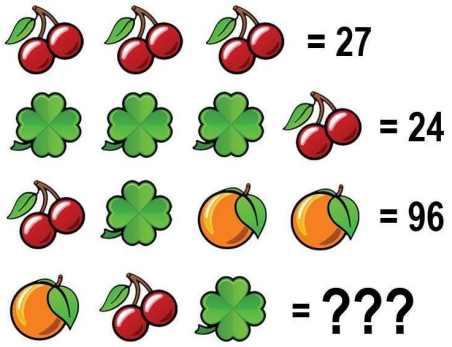
$$63 = \frac{9}{5}C$$

$$63\left(\frac{5}{9}\right) = C$$

$$35 = C$$

Category 5
Algebra
Meet #2 - December, 2016

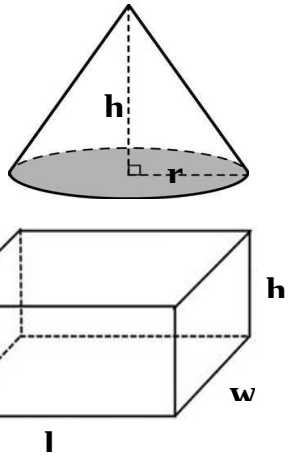
1) This puzzle was circulating throughout the Internet a few weeks ago. What is the value of the last line ?



2) Moe is three times as old now as Larry will be four years from now. If Larry is five years old now, then how many years old will Moe be in seven years?

3) The formula for the total surface area (TSA) of a cone is

TSA = $\pi r^2 + \pi rh$ **where h = the height and r = the radius of the circular base. Find the smallest number of cubic inches in the volume of a rectangular box that tightly contains the cone whose total surface area is** 77π **square inches and the diameter of its circular base is 14 inches. The formula for the volume of a rectangular box is V = l w h where l = the length of the base, w = the width of the base, and h = the height of the box. Assume that the circular base of the cone occupies the same plane as the bottom of the rectangular box.**



Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #2 - December, 2016

Answers	
1)	55 *or* 24
2)	34
3)	784

1) If three cherries = 27, then one cherry = 9. If three clovers and a cherry = 24, then three clovers = 15 and one clover = 5. Two oranges, a clover, and a cherry make 96, so two oranges = 82 and one orange = 41. Finally, one orange, one cherry, and one clover make 41 + 9 + 5, or 55.

Equally valid is a solution where the implied operation is multiplication, yielding one cherry = 3, one clover = 2, one orange = 4, and the final answer is 3 x 2 x 4 = 24.

So, either answer, 55 or 24, is correct.

2) Start with Larry being 5 years old now. Then work to the right. Then go to Moe - now he is 4(9), or 36. Then work to the right. Nyuk nyuk!

	Now	in 4 yrs	in 7 yrs
Moe	3(9) = 27	31	34
Larry	5	9	12

3) Use the formula for the TSA of the cone to find the height of the cone. This will also be the height of the box. The length and width of the box are each the diameter of the cone.

TSA (cone) $= \pi r^2 + \pi rh = \pi(7^2) + \pi(7)(h) = 77\pi$

$\pi(49) + \pi(7)(h) = 77\pi$

Divide both sides by pi : $49 + 7h = 77$

$7h = 28$

$h = 4.$

The length of the rectangular box = the width = 14, and h = 4. The volume of the rectangular box = l w h = (14)(14)(4) = 784 cubic inches.

Category 5
Algebra
Meet #2 - November, 2014

1) If 2A + 6B = 26, then what is the value of 5A + 15B - 10 ?

2) Mutt and Jeff live 100 miles apart. Each rides a bicycle toward the other's house along a straight road connecting the two. Mutt pedals at a constant rate of 8 miles per hour (mph) while Jeff pedals at a constant rate of 12 miles per hour. If Mutt and Jeff leave their houses at the same time, then how many miles are they from Jeff's house when they meet ?

***Footnote*: Mutt and Jeff were characters in a long-running comic strip by the same name from 1907 to 1983, published in daily newspapers around the world. For more information: http://en.wikipedia.org/wiki/Mutt_and_Jeff**

3) Rick travelled from Lompoc to Visalia, averaging 60 kilometers per hour (kph). He averaged 90 kilometers per hour on the return trip home, following the same route. What was Rick's average rate of speed, in kilometers per hour, for the entire round trip ?

Answers

1) _____

2) _____

3) _____

Solutions to Category 5
Algebra
Meet #2 - November, 2014

Answers	
1)	55
2)	60
3)	72

1) The expression 5A + 15B is 2.5 times as great as 2A + 5B. So, (2.5)(26) - 10 = 65 - 10 = 55.

2) Let X = the number of hours that each man travels. Then 8X is Mutt's distance and 12X is Jeff's distance. Since they travel toward one another and meet at a point in between their houses, they travel a total of 100 miles.

$$8X + 12X = 100$$
$$20X = 100$$
$$X = 5$$

In 5 hours, Jeff travels (12)(5), or 60 miles, and is therefore 60 miles from his house when they meet.

3) The average rate of speed, in kph, for the entire round trip is equal to the total distance in km divided by the total time in hours. The exact distance between Lompoc and Visalia is irrelevant. For the sake of the numbers in this problem, let's use a multiple of 60 and 90, such as 540 km, as the distance from Lompoc to Visalia. So, 540 / 60 = 9 hours. Also, 540 / 90 = 6 hours.
Therefore, total distance divided by total time

$$= (2)(540) / (6 + 9)$$
$$= 1080 / 15$$
$$= 72 \text{ kph.}$$

Category 5
Algebra
Meet #2, November/December 2012

1. Five consecutive multiples of 17 have a sum of 510. What is the greatest of these five multiples of 17?

2. During the summer of 2012, Joel earned \$27 less than three times as much as he earned during the summer of 2011. For the summer of 2013, he hopes to earn \$27 less than three times his earnings of the summer of 2012. If all goes according to his plan and we know that he earned \$714 during the summer of 2012, how many more dollars will he earn in the summer of 2013 than he did in the summer of 2011?

3. The formula $S = \frac{n \times (a + l)}{2}$ can be used to find the sum S of an arithmetic sequence, where n is the number of terms, a is the first term, and l is the last term. The last term can be computed using the formula $l = a + (n - 1) \times d$, where a and n are the same as above, and d is the common difference between terms. What is the common difference d if the sum of 17 terms is 1479 and the first term is 39?

Answers

1. _____
2. \$ _____
3. _____

Solutions to Category 5
Algebra
Meet #2, November/December 2012

Answers	
1.	**136**
2.	**$1868**
3.	**6**

1. The sum of the five multiples of 17 is five times the value of the middle number. Dividing 510 by 5, we find that the middle number must be 102, which is 6×17. The greatest of the five multiples of 17 must be 102 + 17 + 17 = **136**, which is 8×17**.**

2. During the summer of 2011, Joel must have earned (714 +27) ÷ 3 = 741 ÷ 3 = $247. In the summer of 2013, he hopes to earn $3 \times 714 - 27$ = 2142 – 27 = $2115. The difference is 2115 – 247 = **$1868**.

3. Substituting the known values into the first equation, we get the equation $1479 = \frac{17 \times (39 + l)}{2}$, which we can solve for *l*. We multiply both sides by 2 and get $2958 = 17 \times (39 + l)$. Dividing both sides by 17, we get $174 = 39 + l$. This means the last term must be 174 – 39 = 135. Now we substitute the known values into to the second equation and solve for *d*: $135 = 39 + (17 - 1) \times d$. This simplifies to $96 = 16d$, so *d* = **6**.

Category 5 – Algebra

1. A football thrown at a 45° angle at a speed of $V_{meters/second}$ will travel a horizontal distance of D_{meters}, given by the formula $D = \frac{V^2}{10}$.
   A quarterback throws one ball at a speed of $20_{meters/second}$ and a second ball at a speed of $25_{meters/second}$. How many meters are there between the landing spots of the two balls? *(Both are thrown from the same spot, in the same direction).*

2. The product of three consecutive natural numbers equals fifty-six times their sum. What is the middle number?

3. You lit up a candle at $10:00$ o'clock, and noticed that at $11:00$ o'clock the candle was $\frac{2}{3}$ of the size it was at $10:45$. Assuming the candle burns at a constant rate, at what time will it be gone completely?
   Express you answer in the format HH:MM (Hours:Minutes).

Answers
1. _____
2. _____
3. _____

Solutions to Category 5 - Algebra

Answers
1. $22.5\ or\ 22\frac{1}{2}$
2. 13
3. 11:30

1. Based on our formula, the first ball willt travel a distance of $\frac{20^2}{10} = 40\ meters$, and the second ball will go for $\frac{25^2}{10} = 62.5\ meters$. The difference is $22.5\ meters$.

2. Calling our numbers $x-1, x, x+1$ we can write:
   $(x-1)\cdot x\cdot(x+1) = 56\cdot(x-1+x+x+1)$
   $(x-1)\cdot x\cdot(x+1) = 56\cdot 3\cdot x$
   Dividing both sides by the common factor x we get
   $(x-1)\cdot(x+1) = x^2 - 1 = 168$, the solution to which is 13.
   [Even if you're unsure about the very last step, a little trial and error should help].

3. If it takes M minutes for the candle to burn completely, then by 10:45 the portion of the candle already burnt is $\frac{45}{M}$, and by 11:00 it is $\frac{60}{M}$. So from the question we know that: $\left(1-\frac{60}{M}\right) = \frac{2}{3}\cdot(1-\frac{45}{M})$. To solve, we multiply both sides by M and get $(M-60) = \frac{2}{3}\cdot(M-45)$ and so $M = 90\ minutes$, which brings us to 11:30 .
   Another way to think of this is in terms of the rate-of-burn (how much candle is being consumed per minute). If we call this number r then we can write:
   $(1-60\cdot r) = \frac{2}{3}\cdot(1-45\cdot r)$ to get $r = \frac{1\ candle}{90\ minute}$.
   Note that $r = \frac{1}{M}$

Category 5
Algebra
Meet #2, December 2008

1. The sum of 5 consecutive odd numbers is 105. What is the largest of the 5 numbers?

2. Shandra and Terri are sisters who were both given the same amount of money by their mother. Shandra was able to triple her money doing chores, while Terri spent 6 of her dollars. Shandra now has 4 times as much money as Terri. How many dollars did Terri's mother give her?

3. The sum of the first n natural numbers is known as the n^{th} triangular number. The formula for the n^{th} triangular number is $T_n = \frac{n(n+1)}{2}$. The sum of the first n cubic numbers is equal to the n^{th} triangular number squared. If the sum of the first n cubes is 6084, what is the value of n?

Answers

1. _____
2. _____
3. _____

Solutions to Category 5
Algebra
Meet #2, December 2008

Answers

1. 25

2. 24

3. 12

1. By calling the middle of the five odd numbers x, we can use the equation below to find the middle number

$(x-4)+(x-2)+x+(x+2)+(x+4)=105$

$5x=105 \rightarrow x=21$

If the middle number is 21, then the five numbers are 17, 19, 21, 23, 25.

2. Calling the amount of money Terri and Shandra started with S, we can use the equation $3S=4(S-6) \rightarrow 3S=4S-24 \rightarrow S=24$.

3. The sum of the first *n* cubes is equal to $(T_n)^2$, so the first thing we need to figure out is the square root of 6084. We can estimate this by noticing that $80^2=6400$ so we know the square root is less than 80. Since the number ends in 4 its square root must end in 2 or 8. So the square root of 6084 is either 72 or 78 (since we know it is an integer). $72^2 = 5184$ and $78^2 = 6084$. We now know that $T_n = 78 = \frac{n(n+1)}{2} \rightarrow 156 = n(n+1) \rightarrow 12 \cdot 13 = n(n+1) \rightarrow 12 = n$