Sinusoidal Functions as Mathematical Models

1. You are riding a Ferris Wheel that is 40 feet in diameter. Using a stopwatch, you begin timing when you are at the bottom, 3 feet above the ground. You find it takes 8 seconds to go around one time.

a. Sketch the graph of your height vs. time for one full cycle. Label the maximum and minimum points as well as those on the sinusoidal axis.

\[ y = -20 \cos \left( \frac{\pi}{4} x \right) + 20 \]

b. Please find an equation that fits this curve. (either sin/cos)

\[ y = a \sin (b(x + c)) + d \quad y = a \cos (b(x + c)) + d \]

\[ y = -20 \cos \left( \frac{\pi}{4} x \right) + 20 \]

\[ \text{amp} \ 20 \]

\[ \text{pa} \ 8 \]

\[ \text{va} \ \text{up} \ 20 \]

c. Predict your height above the ground when \( t = 6 \) seconds.

\[ y = -20 \cos \left( \frac{\pi}{4} \cdot 6 \right) + 20 = 20 \]

2. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight first reaches a high point of 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 sec.

a. Sketch the graph of distance vs. time for one full cycle. Label the maximum and minimum points as well as those on the sinusoidal axis.

b. Please find an equation that fits this curve. (either sin/cos)

\[ y = a \sin (b(x + c)) + d \quad y = a \cos (b(x + c)) + d \]

\[ \text{pa} = 3 = \frac{2\pi}{b} \quad b = \frac{2\pi}{3} \]

\[ \text{Amp} = 20 \]

\[ v_g = 50 \]

\[ y = 10 \cos \left( \frac{2\pi}{3} (x - 0.3) \right) + 50 \]

\[ y = 10 \sin \left( \frac{2\pi}{3} (x + 0.5) \right) + 50 \]
1. Suppose that the water wheel pictured below rotates at 6 revolutions per minute (rpm). You start your stopwatch. Two seconds later, point P on the rim of the wheel is at its greatest height. Model the distance $d$ of point P from the surface of the water in terms of the number of seconds $t$ the stopwatch reads. Assume that $d$ varies sinusoidally with $t$, sketch a graph, write an appropriate equation and predict the distance above the water line that point P will be after 5.5 seconds.

$$y = 7 \cos \left( \frac{\pi}{5} (x - 2) \right)$$

2. As you ride a Ferris wheel, your distance from the ground varies sinusoidally with time. When the last seat is filled and the Ferris wheel starts, your seat is at the position shown in the drawing. Let $t$ be the number of seconds that have elapsed since the Ferris wheel started. You find that it takes you 3 seconds to reach the top, 43 feet above the ground, and that the wheel makes a revolution once every 8 seconds. The diameter of the wheel is 40 feet. Sketch a graph of the sinusoid, write the particular equation of this sinusoid and predict your height above the ground when: a) $t = 6$ seconds, b) $t = 4\frac{1}{3}$, c) $t = 9$, and d) $t = 0$.

$$y = 20 \cos \left( \frac{\pi}{4} (x - 3) \right) + 3$$

3. As you stop your car at a traffic light, a pebble becomes wedged between the tire treads. When you start off, the distance of the pebble from the pavement varies sinusoidally with the distance you have traveled. The period is, of course, the circumference of the wheel. Assume that the diameter of the wheel is 24 inches. Sketch the graph of this function, write the equation of this function, and predict the distance from the pavement when you have gone 15 inches.

$$y = -12 \cos \left( \frac{\pi}{12} (x) \right) + 12$$
4. Naturalists find that the populations of some kinds of predatory animals vary periodically. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept when time \( t = 0 \) years. A minimum number of 200 foxes, occurred when \( t = 2.9 \) years. The next maximum, 800 foxes, occurred at \( t = 5.1 \) years. Sketch the graph of this sinusoid, write an equation expressing the number of foxes as a function of time, \( t \), and predict the population when \( t = 7 \). Foxes are declared to be an endangered species when their population drops below 300. Between what two values of \( t \) were foxes first endangered?

\[
\begin{align*}
\text{Amp} &= \frac{600}{a} = 300 \\
\text{Period} &= 2(5.1 - 2.9) = 4.4 = \frac{2\pi}{b} \\
b &= \frac{2\pi}{4.4} = 1.428
\end{align*}
\]

\[
y = -300 \cos \left( \frac{1.428(x - 2.9)}{4} \right) + 500
\]

5. A weight attached to the end of a long spring is bouncing up and down. As it bounces, its distance from the floor varies sinusoidally with time. You start your stopwatch. When the stopwatch reads 0.3 seconds, the weight first reaches a high point 60 cm above the floor. The next low point, 40 cm above the floor, occurs at 1.8 seconds. Sketch a graph of this function, find an equation expressing distance from the floor in terms of time, and predict the distance from the floor when the stopwatch reads 17.2 seconds.

\[
y = 10 \cos \left( \frac{2\pi}{3}(x - .3) \right) + 50
\]

\[(17.2, 43.309)\]

6. Suppose that you are on the beach. At 2:00 p.m. the tide is in (water is deepest). At that time you find the depth of the water at the end of the pier is 1.5 meters. At 8:00 p.m. the same day, when the tide is out, you find that the depth of the water is 1.1 meters. Assume that the depth of the water varies sinusoidally with time. Sketch the function, write an equation expressing depth of water in terms of the number of hours that have elapsed since 12:00 noon, and predict the depth of the water at 7:00 a.m. the next morning.

\[
\begin{align*}
\text{Amp} &= 2 \\
\text{Period} &= 2(20-8) = 16 = \frac{2\pi}{b} \\
b &= \frac{2\pi}{16}
\end{align*}
\]

\[
y = -\frac{2}{16} \cos \left( \frac{\pi}{16}(x-8) \right) + 1.3
\]

\[7\text{am (7, 1.47)}\]
HW: Sinusoidal Functions as Mathematical Models

1. Mark Twain sat on the deck of a river steamboat. As the paddlewheel turned, a point on the paddle blade moved in such a way that its distance, d, from the water's surface was a sinusoidal function of time. When his stopwatch read 4 seconds, the point was at its highest, 16 feet above the water's surface. The wheel's diameter was 18 ft. and it completed a revolution every 10 seconds.

   a. Sketch the graph of distance vs. time for one full cycle. Label the maximum and minimum points as well as those on the sinusoidal axis.

   ![Graph](image)

   b. Please find an equation that fits this curve. (either sin/cos)

   \[ y = a \sin (b(x + c)) + d \]

   \[ y = a \cos (b(x + c)) + d \]

   \[ y = 9 \cos \left( \frac{\pi}{5}(x - 4) \right) + 7 \]

2. Naturalists find that the populations of some kinds of predatory animals vary periodically. Assume that the population of foxes in a certain forest varies sinusoidally with time. Records started being kept when time, \( t = 0 \) years. A minimum number, 200 foxes, occurred 2.9 years later. The next maximum, 800 foxes, occurred at \( t = 5.1 \) years.

   a. Sketch the graph of the number of foxes vs. time for one full cycle. Label the maximum and minimum points as well as those on the sinusoidal axis.

   ![Graph](image)

   b. Please find an equation that fits this curve. (either sin/cos)

   \[ y = a \sin (b(x + c)) + d \]

   \[ y = a \cos (b(x + c)) + d \]

   \[ y = -300 \cos \left( \frac{5\pi}{11}(x - 2.9) \right) + 500 \]