Meet #1

Problem Categories for this Meet:

1. Mystery: Problem solving

2. Geometry: Angle measures in plane figures including supplements and complements

3. Number Theory: Divisibility rules, factors, primes, composites

4. Arithmetic: Order of operations; mean, median, mode; rounding; statistics

5. Algebra: Simplifying and evaluating expressions; solving equations with 1 unknown including identities
Geometry Information you need to know...

- **Supplements**: Two angles that add up to 180°

- **Complements**: Two angles that add up to 90°

- A *regular polygon* has all sides and all angles of equal measure.

- A straight angle measures 180°

- Angles are named with their vertex in the middle, or just by their vertex. For example,

  ![Diagram](image)

  is named \( \angle ABC \) or just \( \angle B \).

- **Vertical angles** are congruent (equal). In the diagram below, \( \angle ABC \) and \( \angle DBE \) are vertical angles. \( \angle ABD \) and \( \angle CBE \) are also vertical angles.

  ![Diagram](image)

- If two lines are parallel, their **corresponding angles** are congruent. In the diagram below, corresponding angles are labeled with the same number.

  ![Diagram](image)
Any polygon has a sum of its interior angles equal to $180(n - 2)$ with $n$ being the number of sides of the polygon.

For example, a triangle has $180(3 - 2)$ or $180^\circ$; a quadrilateral $180(4 - 2)$ or $360^\circ$; a pentagon $180(5 - 2)$ or $540^\circ$, etc.

To find the measure of an interior angle of a regular polygon, use the formula $\frac{180(n - 2)}{n}$ with $n$ being the number of sides of the polygon. **Interior angles** are the angles indicated by arcs in the following diagram.

To find the measure of an exterior angle of a regular polygon, simply divide 360 by the number of sides of the polygon. **Exterior angles** are the angles indicated by arcs in the following diagram. Formula: $360 \div n$
Category 2
Geometry
Meet #1, October 2003

1. Lines $\overline{TP}$, $\overline{BG}$, and $\overline{DM}$ intersect at point $O$. $m\angle BOT = 47$ degrees and $m\angle MOG = 29$ degrees. How many degrees are in the measure of angle $DOP$?

2. Lines $m$ and $n$ are parallel. $m\angle HIJ = 148$ degrees and $m\angle QRS = 133$ degrees. How many degrees are in the measure of angle $IJK$ if it is less than 180 degrees?

3. The sum of the supplement of angle $A$ and the complement of angle $A$ measures sixteen degrees more than a straight angle. How many degrees are in the measure of angle $A$?

Answers
1. _______________
2. _______________
3. _______________

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Solutions to Category 2
Geometry
Meet #1, October 2003

Answers

1. The measures of angles $BOT$, $TOM$, and $MOG$ must add up to 180 degrees since $O$ is a point on line $BG$. Thus the measure of angle $TOM$ must be $180 - 47 - 29 = 104$ degrees. Angles $TOM$ and $DOP$ are verticle angles and therefore have the same measure. The measure of angle $DOP$ is 104 degrees.

2. Angle $HIJ$ measures 148 degrees, so angle $JIK$ must measure $180 - 148 = 32$ degrees. Angle $QRS$ and angle $IKR$ are corresponding angles, so they have the same measure. This means angle $JKI$ must measure $180 - 133 = 47$. The total angle sum of triangle $IJK$ has to be 180 degrees, so angle $IJK$ must measure $180 - 32 - 47 = 101$ degrees.

3. The supplement of angle $A$ measures $180 - A$. The complement of angle $A$ measures $90 - A$. Their sum is $(180 - A) + (90 - A) = 270 - 2A$. If this amount is sixteen degrees more than a straight angle, then we can write the equation $270 - 2A = 180 + 16$ and solve for $A$.

\[
\begin{align*}
270 - 2A &= 180 + 16 \\
270 - 2A &= 196 \\
270 &= 196 + 2A \\
270 - 196 &= 2A \\
74 &= 2A \\
A &= 37
\end{align*}
\]
Category 2
Geometry
Meet #1, October 2005

1. Tim added $x$ degrees to a 27-degree angle. The complement of this new angle was 48 degrees. He then added $y$ degrees to this 48-degree angle. The complement of this new angle was 7 degrees. Find the value of $x + y$.

2. In the figure at right, the measure of angle ABC is 60 degrees, the measure of angle DEF is 100 degrees, and the measure of angle DGI is 116 degrees. How many degrees are in the measure of angle FHG?

3. The sum of the complement of angle $x$ and the supplement of angle $x$ is 10 degrees less than eight times the angle $x$. How many degrees are in the measure of angle $x$?

<table>
<thead>
<tr>
<th>Answers</th>
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<tbody>
<tr>
<td>1. ______</td>
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<tr>
<td>2. ______</td>
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<tr>
<td>3. ______</td>
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</tbody>
</table>

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Solutions to Category 2
Geometry
Meet #1, October 2005

Answers

1. The measures of two complementary angles add up to 90 degrees. Since $27 + 48 = 75$, the unknown amount $x$ must have been $90 - 75 = 15$ degrees. Likewise, since $48 + 7 = 55$, $y$ must be $90 - 55 = 35$. The value of $x + y$ is thus $15 + 35 = 50$.

2. Vertical angles are congruent, straight angles have a sum of 180 degrees, and triangles have an angle sum of 180 degrees. Using these three facts, the angles of the two small triangular regions can be determined from the angle measures given. The measure of angle FHG is 24 degrees.

3. The complement of angle $x$ is $90 - x$ and the supplement of angle $x$ is $180 - x$. Their sum is $(90 - x) + (180 - x) = 270 - 2x$. We know that this sum is equal to ten less than eight times angle $x$, or $8x - 10$. Now we can write an equation and solve for $x$.

\[
\begin{align*}
270 - 2x &= 8x - 10 \\
+10 &= +10 \\
280 - 2x &= 8x \\
+2x &= +2x \\
280 &= 10x \\
x &= 28
\end{align*}
\]

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1. In the diagram at the left, the four segments: \( \overline{MQ}, \overline{NR}, \overline{OS}, \) and \( \overline{PT} \) all intersect at the same point creating 8 angles labeled with either their angle measures or a variable name. What is the value of \( a + b + c + d + g \)?

2. Two angles have a sum of 170 degrees. The complement of one of the angles is equal to the supplement of the other. How many degrees are in the measure of the larger angle?

3. In the diagram to the right, \( \overline{AB} \parallel \overline{DC} \parallel \overline{FE} \) and \( \overline{BC} \parallel \overline{DE} \parallel \overline{GH} \parallel \overline{AK} \). The measure of angle ABC is 73 degrees and the measure of angle FGH is 111 degrees. What is the measure of angle EFG? (the symbol \( \parallel \) means "is parallel to")

**Answers**

1. ________________
2. ________________
3. ________________
Solutions to Category 2
Geometry
Meet #1, October 2007

Answers

1. Since there is a total of 360 degrees around a point, all the angles in the diagram must add up to 360 degrees.

2. 130

So \( a + b + c + d + g + 35 + 48 + 61 = 360 \rightarrow g + 144 = 360 \rightarrow g = 216 \).
Therefore the sum of \( a, b, c, d \) & \( g = 216 \) degrees.

2. The complement of an angle \( x \) is \( 90 - x \) and the supplement of an angle \( y \) is \( 180 - y \). If the complement of one equals the supplement of the other \( 90 - x = 180 - y \rightarrow y - x = 90 \). That means the difference of the two angles is 90 degrees. Since we know the sum is 170 and one of them is 90 more than the other, simple algebra or guess and check can find that 130 & 40 fit both those requirements and \( 130 \) is the larger number.

3. Since AB and DC are parallel, the same side interior angles(ABC and BCD) have a sum of 180 degrees. So angle BCD has a measure of 107 degrees. Angle BCD is alternate interior to EDC which is alternate interior to DEF, so angle BCD has the same measure as DEF(107 degrees). Angle AFE and DEF are same side interior angles they have a sum of 180 degrees and angle AFE has a measure of 73. Since AFG is alternate interior with FGH, AFG has a measure of 111 degrees. Since angles AFE and EFG form angle AFG, their sum is 111 degrees. So :

\[
AFE + EFG = 111 \rightarrow 73 + EFG = 111 \rightarrow EFG = 38
\]

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Category 2 - Geometry
Meet #1, October 2009

1. In the above diagram, right triangle ABC and equilateral triangle BCD intersect at point E. Given the angle measurements above, how many degrees in the measure of \(\angle BEC\)?

2. The supplement to an angle \(x\) is two-and-a-half times its complement. How many degrees in the measure of \(x\)?

3. How many degrees in the measure of angle \(Z\)?

Answers
1. 
2. 
3. 

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1. $\angle CBD=60$ degrees, therefore $\angle DBA=30$ degrees, therefore $\angle BEA =105$ degrees, and so $\angle BEC$ equals 75 degrees.

Another way to see this: $\angle EBC=60$ degrees, $\angle BCE=45$ degrees, therefore $\angle CEB=75$ degrees.

2. We can write the equation $(180 - x) = 2.5 \cdot (90 - x)$ and solve: $1.5 \cdot x = 0.5 \cdot 90$ and so $x = 30$ degrees.

3. By completing triangle CBO to 180 degrees we get $\angle BOC=24$ degrees.

By completing around O to 360 degrees we get $\angle BOA=46$ degrees.

Complete triangle ACO to 180 to get $\angle OAC=54$ degrees.

Finally $Z$ supplements it, so is 126 degrees.

<table>
<thead>
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<tbody>
<tr>
<td>1. 75</td>
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<td>2. 30</td>
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<tr>
<td>3. 126</td>
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Category 2 – Geometry

1. The supplementary angle to a given $\angle A$ measures 4 times as much as $\angle A$’s complementary angle. How many degrees are there in the measure of $\angle A$?

2. In the drawing below, the measure of one angle is given as 60 degrees, and the measure of the angle ‘$x$’ is half as much as the measure of the angle ‘$y$’. How many degrees are there in the measure of the angle ‘$z$’?

   ![Diagram](image)

3. The angle between the hour-hand and the minute-hand on a clock at 3 o’clock is 90 degrees. What is it 20 minutes earlier? (*Measure from the hour hand to the minute-hand counter-clockwise*).

   ![Diagram](image)

**Answers**

1. _________ degrees

2. _________ degrees

3. _________ degrees
Solutions to Category 2 – Geometry

1. If we write this information algebraically, then we know that:
   \[180 - A = 4 \cdot (90 - A)\]. This we can simplify to:
   \[180 - A = 360 - 4 \cdot A\] and rearrange to: \[3 \cdot A = 180\] to find \(A = 60\).
   Alternatively, we can engage in a little trial and error to find out the value of \(A\).

2. The missing angle in the triangle equals \(x\) (opposite angles), and so
   we know that \(x + y + 60 = 180\) as the three angles in a triangle. Given that \(x\)
   is half as much as \(y\), we can rewrite this as \(3 \cdot x + 60 = 180\) to find that
   \(x = 40\) degrees. \(z\) is the supplement to \(x\) and so equals 140 degrees.

3. It takes the minute hand an hour to complete a revolution, so 20 minutes earlier,
   it was 120 degrees back (pointing at the numeral 8). The hour hand progresses
   at a pace of only 30 degrees every hour, so in 20 minutes it only progresses 10
   degrees. The answer then is \(90 + 120 - 10 = 200\) degrees.