Problem Categories for this Meet:

1. Mystery: Problem solving
2. **Geometry: Angle measures in plane figures including supplements and complements**
3. Number Theory: Divisibility rules, factors, primes, composites
4. Arithmetic: Order of operations; mean, median, mode; rounding; statistics
5. Algebra: Simplifying and evaluating expressions; solving equations with 1 unknown including identities
Important Information you need to know about GEOMETRY:
*Solid Geometry (Volume and Surface Area)*

Know these formulas!

<table>
<thead>
<tr>
<th>SHAPE</th>
<th>SURFACE AREA</th>
<th>VOLUME</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rect. prism</strong></td>
<td>2(LW + LH + WH)</td>
<td>LWH</td>
</tr>
<tr>
<td><strong>Any prism</strong></td>
<td>sum of areas of all surfaces</td>
<td>H(Area of Base)</td>
</tr>
<tr>
<td><strong>Cylinder</strong></td>
<td>$2\pi R^2 + 2\pi RH$</td>
<td>$\pi R^2 H$</td>
</tr>
<tr>
<td><strong>Pyramid</strong></td>
<td>sum of areas of all surfaces</td>
<td>1/3 H(Base area)</td>
</tr>
<tr>
<td><strong>Cone</strong></td>
<td>$\pi R^2 + \pi RS$</td>
<td>1/3 $\pi R^2 H$</td>
</tr>
<tr>
<td><strong>Sphere</strong></td>
<td>$4\pi R^2$</td>
<td>$4/3 \pi R^3$</td>
</tr>
</tbody>
</table>

**Surface Diagonal:** any diagonal (NOT an edge) that connects two vertices of a solid while lying on the surface of that solid.

**Space Diagonal:** an imaginary line that connects any two vertices of a solid and passes through the interior of a solid (does not lie on the surface).
1) Melinda unwraps a gift contained in a cube-shaped box that has a volume of 729 cubic inches. How many square inches of wrapping paper are on the surface of the box (all sides)?

2) A cylindrical box of Earthquake Oats is 28 centimeters tall. Its circular top has a diameter of 12 centimeters. Wyatt puts 125 cubic centimeters of oats into individual tight-lock bags for his daily breakfasts. All oats must be in a bag, even if the last bag is not full. How many bags in all does Wyatt need in order to store a boxful of oats?

3) It takes 21 seconds to inflate a spherical balloon to a diameter of 8 inches. How long should it take to inflate a similar balloon to a diameter of 40 inches if it is inflated at the same rate as for the smaller balloon? If the answer is B minutes and C seconds, then give the value of B if C is less than 60.

**ANSWERS**

1) ________ sq. in.

2) ________ bags

3) ________ = B
Solutions to Category 2
Geometry
Meet #5 - March, 2014

<table>
<thead>
<tr>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1) 486</td>
</tr>
<tr>
<td>2) 26</td>
</tr>
<tr>
<td>3) 43</td>
</tr>
</tbody>
</table>

1) Take \(\sqrt[3]{729}\) to find the length of one edge = 9 inches. Each face has an area = 9 x 9, or 81 square inches. Since there are six surfaces, the surface area is 6 x 81, or 486 square inches.

2) Since the formula for the volume of a cylinder depends on its radius, take half of the 12 cm diameter so that the radius is 6 cm long.

Volume of cylinder = \(\pi r^2h = \pi(6^2)(28) \approx 3166.72\) cubic cm.

Divide the volume by 125 to find the number of bags needed:

\[
\frac{3166.72}{125} = 25.3\text{ bags. So, 26 bags are required to hold all of the oats.}
\]

3) The simplest approach requires this knowledge: The ratio of the volumes of two similar objects is equal to the cube of their linear ratio. (Also, while we are at it: The ratio of the surface areas of two similar objects is equal to the square of their linear ratio.)

Dividing the two diameters, 40 / 8, yields the fact that the larger balloon is five times the diameter of the smaller balloon, or 125 (five cubed) times its volume.

Multiply 21 by 125 to find the number of seconds required to inflate the larger balloon = 2625 seconds.

Divide 2625 by 60 to convert the time to minutes = 43.75 minutes = 43 minutes and 45 seconds. Since the question asks for the value of \(B\), the number of minutes, then \(B = 43\).

A more popular approach would be to calculate the two volumes and then divide to find how many times larger the big balloon is:

\[
\frac{\frac{4}{3}\pi(20^3)}{\frac{4}{3}\pi(4^3)} = \frac{\frac{4}{3}\pi(8000)}{\frac{4}{3}\pi(64)} = \frac{8000}{64} = 125.
\]

The remainder of the solution would be the same as for the former solution.
Category 2 – Geometry

*Use \( \pi = 3.14 \) whenever necessary*

1. How many cubes that are 1" \( \times \) 1" \( \times \) 1" have a combined surface area as a cube that is 8" \( \times \) 8" \( \times \) 8" ?

2. How many spherical raindrops with a diameter of 2 millimeters will it take to fill a cylindrical bucket whose radius is 314 millimeters and height is 400 millimeters?

3. A cylinder is built around a ball (see diagram below) so that they have the same radius and that the cylinder’s height equals the ball’s diameter. If the ball’s volume is 200 cubic centimeters (cc), then how many cc are there in the cylinder’s volume?

![Diagram of a cylinder and a ball](https://via.placeholder.com/150)

**Answers**

1. _______________

2. _______________

3. ____________ cc

www.imlem.org
Solutions to Category 2 – Geometry

1. A cube has 6 faces, so its surface area is 6 times the area of a face. The little cube’s face has an area of 1 square inch, and the large cube has a face with an area of 64 square inch, so it will take 64 little cubes to match the surface area of the larger one (and, of course, $8^3$ cubes to match the volume).

2. The volume of a raindrop is $\frac{4}{3} \cdot \pi \cdot 1^3$ cubic millimeter.

   (The radius of the drop is 1 millimeter)

   The volume of the bucket is $\pi \cdot 314^2 \cdot 400$ cubic millimeter and so it will take:

   \[
   \frac{\pi \cdot 314^2 \cdot 400}{\frac{4}{3} \pi} = \frac{3 \cdot 314^2 \cdot 400}{4} = 29,578,800
   \]

   raindrops to fill.

3. A ball’s volume is given by $V_{ball} = \frac{4}{3} \cdot \pi \cdot R^3$ and a cylinder’s volume is given by $V_{cylinder} = \pi \cdot R^2 \cdot H$.

   In our case the radii are the same, and $H = 2 \cdot R$ and so $V_{cylinder} = 2 \cdot \pi \cdot R^3$ or in other words $V_{cylinder} = \frac{3}{2} \cdot V_{ball} = 300 \text{ cc}$
Category 2 - Geometry
Meet #5, March 2010

1. How many square feet are in the surface area of a cube with a volume of 8 cubic feet?

2. A ball with a radius of 3 inches has the same volume as a pyramid with a square base measuring \(6 \times 6\) inches. How many inches are in the pyramid’s height?
   
   Use \(\pi = 3.14\) and express your answer as a decimal to the nearest hundredth.

3. Cylinder A has a radius of 3 inches, and is filled with water to a height of 5 inches. When connected by a small tube to cylinder B (which is initially empty) with a radius of 4 inches, according to the laws of physics, water will flow out of A and into B until the level (height) of water in both is the same. How many inches will the new height be?
   
   Express your answer as a decimal to the nearest tenth.
   
   [We ignore the volume of water in the connecting tube].

---

### Answers

1. ____________
2. ____________
3. ____________
You may use a calculator today!

Solutions to Category 2 - Geometry

Meet #5, March 2010

<table>
<thead>
<tr>
<th>Answers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 24</td>
</tr>
<tr>
<td>2. 9.42</td>
</tr>
<tr>
<td>3. 1.8</td>
</tr>
</tbody>
</table>

1. A cube’s volume is $L^3$ where $L$ is its side’s length. Therefore our cube is 2 feet long in each direction. Its surface is made of six squares measuring $2 \times 2$ feet each, or 4 square feet each, so the total is $6 \cdot 4 = 24$ square feet.

2. The ball’s volume is $V = \frac{4}{3} \pi R^3 = \frac{4}{3} \cdot 3.14 \cdot 3^3 = 36 \cdot 3.14 = 113.04$ cubic inches. The pyramid’s volume, which is the same, equals $\frac{1}{3} \cdot 6 \times 6 \cdot H$ where $H$ is its height.

We get that $H = \frac{3 \cdot V}{6 \cdot 6} = \frac{3 \cdot 113.04}{36} = 9.42$ inches.

3. The idea is to understand the the same volume of water – $V$ – that initially fills cylinder A to a height of 5 inches, will be filling both A and B to a new height which we’ll call $H$.

A cylinder’s volume is given by the formula $V = \pi R^2 H$ and so the volume of water to begin with is $V = \pi \cdot 3^2 \cdot 5 = 45 \cdot \pi$ cubic inches. This same volume will now be spread in both cylinders to a new height $H$, so we can write

$V = 45 \cdot \pi = \pi \cdot H \cdot (3^2 + 4^2)$ and so we get $H = \frac{45}{3^2+4^2} = \frac{45}{25} = 1 \frac{4}{5} = 1.8$ inches.
1. Twenty-two unit cubes were used in building the “U” shape at the right. How many square units are in the surface area of this figure? (Note: The back view is the same as the front.)

2. How many square feet are in the surface area of the right trapezoidal prism below? (diagram not drawn to scale)

3. The diagram to the right shows a hemisphere sitting on top of a cone. The circular base of the hemisphere and the circular base of the cone have the same area. The radius of the hemisphere is 6 cm and the slant height of the cone is 10 cm. How many cubic centimeters are in the volume of the combined shape? Use 3.14 as an estimation of \( \pi \) and express your answer to the nearest tenth.

**Answers**

1. _______________
2. _______________
3. _______________
Solutions to Category 2
Geometry
Meet #5, March 2008

Answers

1. 70

2. 1048

3. 753.6

1. Viewing from the front there are 11 visible squares, and same from the back. From both the left and right sides there are 8 viewable squares. From both the top and bottom there are 10 viewable squares. There are an additional 12 viewable squares inside on the left and right faces of the “U” shape. That’s a total of $11 + 11 + 8 + 8 + 10 + 10 + 12 = 70$ square units.

2. There are 6 faces to this prism. The top has area $10 \times 20 = 200$. The bottom has area $18 \times 20 = 360$. The left face has area $6 \times 20 = 120$. The right face has area $10 \times 20 = 200$. The two trapezoids have area $\frac{(10 + 18) \times 6}{2} = 84$. The total surface area is $200 + 360 + 120 + 200 + 84 + 84 = 1048$.

3. The volume of the hemisphere is $\frac{1}{2} \times \frac{4}{3} \pi 6^3 = \frac{2}{3} \pi 216 = 144\pi \approx 452.16$.

Use the Pythagorean Theorem to find that the height of the cone is 8 cm. So the volume of the cone is $\frac{1}{3} \pi 6^2 \times 8 = \frac{1}{3} \pi 36 \times 8 = \pi 96 \approx 301.44$. The combined volume is $452.16 + 301.44 = 753.6$. 
Category 2
Geometry
Meet #5, March 2006

1. The figure at right is made from 8 unit cubes that are glued together. There is a hole through the middle of the object. How many square units are in the surface area of the entire figure?

2. A plastic sphere with an radius of 10 cm is full of water. If the water from this sphere is poured into a cylinder that has a radius of 10 cm and a height of 20 cm, what fraction of the cylinder will be filled with water? Disregard the thickness of the plastic and express your answer as a simplified fraction.

3. The base of the prism on the left is a square with a side length of 1 unit. The base of the prism on the right is a right triangle with legs of length 1 unit. Both prisms are 2 units long. What is the difference between the surface area of the square based prism on the left than the surface area of the triangular based prism on the right? Express your answer as a decimal to the nearest thousandth of a square unit. (Note that what is called the base may not be on the bottom.)

Answers
1. _______________
2. _______________
3. _______________
Solutions to Category 2
Geometry
Meet #5, March 2006

Answers

1. 32
2. $\frac{2}{3}$
3. 2.172

1. There are 8 unit squares on the front and 8 unit squares on the back of the figure. There are 12 unit squares around the outer rim and 4 unit squares around the inner rim. The total is thus $8 + 8 + 12 + 4 = 32$ unit squares.

2. The particular radius of the sphere doesn’t matter, as long as the height of the cylinder is equal to two radii, which is the diameter of the sphere. In general, the volume of the sphere is $\frac{4}{3}\pi r^3$, and the volume of the cylinder is $\pi r^2 h = \pi r^2 2r = 2\pi r^3$. The ratio of their volumes is thus
$$\frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{4}{3} \div 2 = \frac{2}{3},$$
so the cylinder will be $\frac{2}{3}$ full of water.

3. The square based prism on the left has six faces: two that are unit squares and four that are 1 by 2 rectangles. The surface area of the square based prism is thus $2 \times 1 \times 1 + 4 \times 1 \times 2 = 10$ square units. The triangular based prism on the right has five faces: two that are isosceles right triangles with legs of 1 unit, two that are 1 by 2 rectangles, and one that is a rectangle with a width of $\sqrt{2}$ and a length of 2. The surface area of the triangular prism is $2 \times \frac{1}{2} \times 1 \times 1 + 2 \times 1 \times 2 + 1 \times \sqrt{2} \times 2$ $= 5 + 2\sqrt{2} \approx 7.828$ square units. The desired difference is $10 - 7.828 = 2.172$. 
Category 2
Geometry
Meet #5, April 2004

1. How many space diagonals are there in a hexagonal prism such as the one shown at right? A space diagonal is an imaginary line that connects any two vertices of a solid and passes through the interior of the solid.

2. A roll of toilet paper has a diameter of 12 centimeters and a height of 11.4 centimeters. The inner diameter of the tube is 4 centimeters. How many cubic centimeters are in the volume of the solid part of the roll of toilet paper? Use 3.14 for Pi and round your result to the nearest tenth.

3. One cubic centimeter of water is equal to one milliliter of water. This means that a cubic decimeter, such as the one shown below, can hold one liter of water when it is filled to the brim. A steel sphere is dropped into a full cubic decimeter, displacing some water, which spills over the sides. When the ball is removed, it is noted that 732 milliliters of water remain in the cube. How many centimeters are in the diameter of the sphere? Use 3.14 for π and round your answer to the nearest whole number.

Answers
1. ____________
2. ____________
3. ____________
Solutions to Category 2  Average team got 9.6 points, or 0.8 questions correct
Geometry
Meet #5, April 2004

Answers

1. 18

Three space diagonals can be drawn from each vertex of each base of the prism to vertices on the other base.

2. 1145.5

If we imagine drawing all of these, however, we will be drawing every space diagonal twice or double counting. Therefore, we should imagine drawing all possible space diagonals from just one of the bases. There are \(6 \times 3 = 18\) space diagonals in a hexagonal prism.

2. We will compute the volume of the roll of toilet paper as if there were no hollow tube through it. Then we will subtract the volume of this hollow tube. The formula for the volume of a cylinder is \(V_{cylinder} = A_{base} \times h = \pi r^2 h\). The diameter of the roll is 12 cm, so the radius is 6 cm. Using \(r = 6\), \(h = 11.4\) and \(\pi = 3.14\) in the formula above, we get:

\[
V_{full\ cylinder} = 3.14 \cdot 6^2 \cdot 11.4 = 1288.656
\]

Similarly, we compute the volume of the hollow tube, using \(r = 2\) (half of 4), \(h = 11.4\) and \(\pi = 3.14\):

\[
V_{hollow\ tube} = 3.14 \cdot 2^2 \cdot 11.4 = 143.184
\]

Finally, subtracting the volume of the tube from the volume of the full cylinder, we get:

\[
V_{roll} = V_{full\ cylinder} - V_{hollow\ tube} = 1288.656 - 143.184 = 1145.472
\]

Rounding this to the nearest tenth, we get 1145.5 cubic centimeters.

3. Since 732 milliliters remain in the cube, the steel ball must have displaced \(1000 - 732 = 268\) milliliters. Since 1 m\(^3\) = 1 cm\(^3\), we know that the volume of the sphere is 268 cubic centimeters. Substituting \(V = 268\) and \(\pi = 3.14\) into the formula \(V_{sphere} = \frac{4}{3} \pi r^3\), we can solve for \(r\) as follows:

\[
268 = \frac{4}{3} \cdot 3.14 \cdot r^3 \Rightarrow r^3 = 268 \cdot \frac{3}{4} \cdot \frac{1}{3.14} \Rightarrow r^3 = 64.01273885 \Rightarrow r \approx 3.99 \Rightarrow r \approx 4
\]

Since \(r = 4\) cm, the diameter of the steel sphere must be 8 centimeters. This may seem too large, but we should note that a cube measuring 8 cm by 8 cm by 8 cm would occupy only \(8^3 = 512\) cubic centimeters or a little more than half of the space.